Centre for Climate and Energy Analyses

# THE EVALUATION OF POLICY IMPACTS - CLIMATE AND AGRICULTURE MODEL EPICA

## TECHNICAL DOCUMENTATION FOR THE MODEL VERSION 2.0

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# AUTHORS AND COPYRIGHT

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# List of abbreviations

| | |
|---|---|
| **CAKE** | Centre for Climate and Energy Analyses |
| **CDR** | Agricultural Advisory Centre |
| **CGE model** | Computable General Equilibrium model |
| **$CH_4$** | Methane |
| **$CO_2$** | Carbon Dioxide |
| **$CO_2$eq** | Carbon Dioxide Equivalent |
| **CU** | Currency Unit |
| **GWP** | Global Warming Potential |
| **EC** | European Commission |
| **EFA** | Ecological Focus Area |
| **EPICA model** | Evaluation of Policy Impacts - Climate and Agriculture Model |
| **ESD** | Effort Sharing Decision |
| **ESR** | Effort Sharing Regulation |
| **EU** | European Union |
| **EU ETS** | European Union Emissions Trading System |
| **EU non-ETS** | Sectors not covered by the European Union Emissions Trading System |
| **EU28** | European Union of 28 Member States |
| **LULUCF** | Land Use, Land Use Change and Forestry |
| **FADN** | Farm Accountancy Data Network |
| **GAMS** | General Algebraic Modelling System |
| **GHG** | Greenhouse Gases |
| **Gt** | Gigatonne |
| **GUS** | Statistics Poland |
| **IPCC** | Intergovernmental Panel on Climate Change |
| **K** | Potassium |
| **KOBiZE** | The National Centre for Emissions Management |
| **kWhe** | Kilowatt-Hours Electric |
| **LU, LSU** | Livestock Unit |
| **MJ** | Megajoule |
| **NEL** | Net Energy Lactation |
| **$N_2O$** | Nitrous Oxide |
| **N** | Nitrogen |
| **P** | Phosphorus |
| **UAA** | Utilised Agricultural Area |
| **UNFCCC** | United Nations Framework Convention on Climate Change |

**Keywords:** climate policy, agricultural policy, greenhouse gas emissions, agriculture modelling, agricultural production analysis, farming practices, non-ETS.

# Introduction

1. Agriculture is one of key contributors to climate change through greenhouse gases being emitted along its production activities, affecting the local environment and global climate (IPCC 2019). Overall it emits a variety of GHG, primarily the nitrous oxide ($N_2O$), methane ($CH_4$) and carbon dioxide ($CO_2$). Main agricultural activities driving the GHG emissions include the production of crops (nitrous oxide from use of fertilisers on agricultural soils to most extent) and livestock (mainly methane from enteric fermentation of live animals and manure management). At the same time agriculture is crucial to ensure food provision to the society by utilising various natural resources such as land and water closely linked to the natural and climate conditions. Complexity of agricultural input towards the GHG emissions creates the necessity to model the current state and estimate the influence of implemented and potential policies dealing with agricultural production as well as changes in agri-food supply.

2. While there is a clear understanding of the need to mitigate the GHG emissions from agriculture, it has to be achieved in a balanced manner in order to maintain the delivery of key functions of agricultural sector. In order to pursue farther reduction, specific measures introduced through systems of incentives or taxation are required, which would encourage farmers and businesses in agricultural sector to invest in technologies and implement production practices that are more efficient in terms of environmental protection and mitigation of GHG emissions. Implementing such measures need to be based on reliable evidence regarding their potential effects, requiring application of modelling tools.

3. To define the type of impact and quantify its magnitude in regard of specific policy measures, various modelling approaches are being used. Structural economic models either of general or partial equilibrium nature have been widely applied for analysis of agricultural policies. While general equilibrium models provide a vast set of data covering the whole economy, partial equilibrium models typically provide rather deep and technical representation of specific sector (while largely neglecting its ties to the rest of the economy, yet still describing both supply and demand side of the sector).

4. In order to assure the most comprehensive approach to policies' modelling and assessment of their impact upon the GHG emissions from agricultural sector, the EPICA model has been developed as a complex approach combining partial equilibrium with linear farm activity optimisation. This enables to model not only the overall sectoral response to external shocks, but also corresponding changes in production (supply) behaviour on the farm micro level based on its income maximisation assumption.

5. As agricultural sector manifests noticeable share in GHG emissions there are other measures, apart from changing production technologies, that could be additionally

utilised to mitigate the emissions, such as production of agri-biogas or pursuing carbon sequestration through such measures as afforestation of agricultural land, conversion of peatlands to paludiculture and other.

## 1. The EPICA model

6. **EPICA: "Evaluation of Policy Impacts – Climate and Agriculture"** is a model aimed at estimation and support of analyses of climate policy inflicted changes in agricultural production (including farm structure, farm practices and carbon sequestration measures) with estimation of its influence upon climate change through greenhouse gas emissions. The model was built to consider wide range of policy instruments, but the essential ones are: direct (as in the current ETS scheme) and indirect (additional tax on selected inputs, such as fertilisers) emission tax, emission quota at farm or sector level and wide range of operational subsidies in line with the ones currently implemented within the Common Agricultural Policy. The model also is able to estimate impact of climate change upon agricultural sector, however it requires the introduction of exogenous parameters (e.g., yield change due to climate change).

7. The EPICA model is an integral part of the modelling approach developed in the LIFE Climate CAKE PL project, aimed at building a sustainable and comprehensive system of creating and exchanging information and knowledge, supporting the development of cross-sectional impact assessment of various solutions in the field of climate and energy policy. The project's objectives are consistent with supporting the implementation of the EU climate policy, support the implementation of the energy and climate package 2020 and the EU climate policy framework until 2030, also in the perspective of the long-term strategy until 2050. The EPICA model is one of the models developed and currently used within the LIFE Climate CAKE PL project in the National Centre for Emissions Management (KOBiZE), that is a part of the Institute of Environmental Protection – National Research Institute (IOŚ-PIB). The project has been developing an analytical tool-box combining a global general equilibrium model (CGE) d-PLACE[1] and linked sectoral models: MEESA[2] for energy, TR$^3$E[3] for transport and EPICA for agriculture (Figure 1).

8. Key feature enabling the EPICA model to stand out among other modelling approaches is the implemented assumption of farm income driving the farm behaviour in their choice

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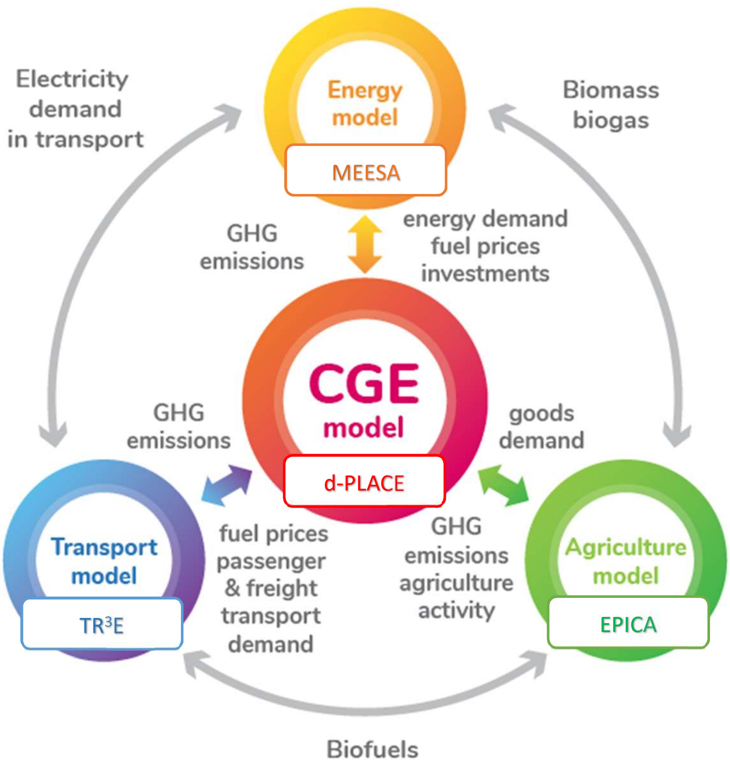
[1] Gąska, J., Pyrka, M., Rabiega, W., Jeszke, R. (2019). The CGE model d-PLACE, Institute of Environmental Protection - National Research Institute / National Centre for Emissions Management (KOBiZE), Warsaw.

[2] Tatarewicz, I., Lewarski, M., Skwierz, S. (2019). The MEESA model documentation, Institute of Environmental Protection - National Research Institute / National Centre for Emissions Management (KOBiZE), Warsaw.

[3] Gąska, J., Rabiega, W., Sikora, P. (2019). The TR3E Model, Institute of Environmental Protection - National Research Institute / National Centre for Emissions Management (KOBiZE), Warsaw.

of production. The choices include the structure of production (referred to as *farm activities*) and production intensity with its relevant processes and practices. The fundamental EPICA model assumption states that the farmers strive to maximise their income by adjusting production structure to the present (expected) market and political situation. Similar approach has been applied by Louhichi et al. (2015) arguing that currently available models have been developed with high aggregation level and are not able to fully capture the impacts of policy measures at farm level.

**Figure 1. LIFE Climate CAKE PL modelling tools**




Source: *CAKE/KOBiZE own study*

9. The EPICA model simultaneously employs several approaches to modelling and combines a partial equilibrium with linear farm activity optimisation programming in order to assure a proper supply-demand balance, as well as provides a highly detailed disaggregation of analysed farm activities. Due to high detail level of agricultural activities the EPICA model and its dataset are currently built to represent solely the agricultural sector of Poland. There is possibility of interaction with the energy model within Life Climate CAKE regarding use of agricultural biomass for energy sources. The baseline dataset implemented in the EPICA model represents year 2015. The choice of Poland as a country to reflect in the EPICA model is substantiated by

the fact that Polish agriculture is one of the major contributors to the agricultural GHG emissions among EU-28 countries (based on 2017 data), being the 6th largest emitter with the share of 7.2% of total EU-28 GHG emissions from agriculture (Eurostat 2018). While a steady decline in GHG from agriculture had been recorded since the 1990 in Poland, from the beginning of new millennia these emissions have been oscillating around the current level, showing only minor annual growth or decline shifts, therefore there's a need to implement policy measures aimed to assure a steady decline in the future.

10. The EPICA model consists of two modules: farm module and market module. The EPICA's farm module is a supply (production) side implemented as a linear programming model calibrated using PMP (Positive Mathematical Programming, Howitt 1995) approach, representing outlined farm types optimising their income subject to resource and technological constraints. Its purpose is to define responses of agricultural sector at the micro-level (the farm) with the ability to capture the policy induced changes in terms of hectares, livestock units, currency units, therefore giving a detailed picture of shifts in particular farm activities, supply of agricultural products and corresponding GHG emissions.

11. The EPICA's market module being a partial equilibrium combines supply from the farm module and demand for products of agricultural origin from the LIFE Climate CAKE PL core CGE model (d-PLACE). The main CGE model employs an aggregated dataset, where all agricultural production is currently represented by the following two sectors: BIO and AGR. The use of aggregated data allows the model to numerically determine the changes of prices for all sectors of the economy in the general equilibrium.

# 2. Farm module

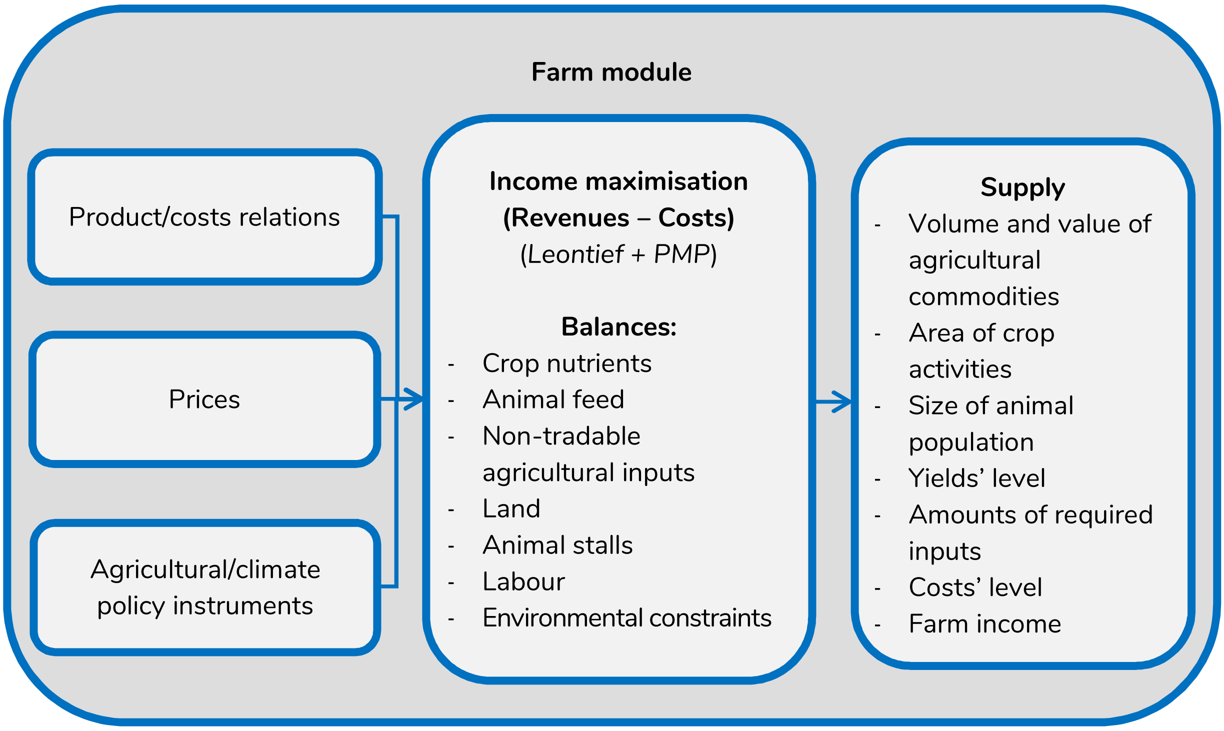
## 2.1. Description and objectives

12. The farm module's objective function is the farm income maximisation constrained by availability of primary production factors and production inputs. The farm module takes into account three key factors being exogenous to particular farm activities (as shown in Figure 2): 1) product/costs relations, 2) price levels, 3) agricultural and climate policy instruments. The main policy instruments considered are the emission taxes, taxes on inputs, emission quotas, emission mitigation subsidies, payments for decreasing or quitting particular production (e.g., cattle payment). These three factors are not under the influence of the farm and there is an objective necessity to adapt to them in the business process. Following these factors, the income maximisation function itself is constrained by several balances, which force it to maintain in realistic boundaries of available resources.

13. The balances implemented in the farm module include: 1) crop nutrients, 2) animal feed (separately for cattle, pigs and poultry), 3) non-tradable agricultural inputs, 4) land, 5) number of animal stalls, 6) labour, and 7) environmental constraints (e.g., maintaining the permanent grassland area above 95% of the current level, according to the CAP requirements).

14. The outcome of the farm module is the updated supply based on the new farm activities' structure. The updated data includes the volume and value of agricultural commodities, area of crop activities, size of animal population, level of yields, amounts of required inputs, level of costs, farm income, and GHG emissions.

**Figure 2. EPICA's farm module operation concept**




Source: *CAKE/KOBiZE own study*

15. The total supply side of agricultural sector is estimated based on the set of farm optimisation submodels, with separate submodel for each production type and size class of farm (defined as *farm type*). There are a total of 19 farm types in the module, reflecting the real farm structure of Poland based on statistical data. Main output of the farm module is a structure of farm activities for each of the defined farm types. Based on it the other data (e.g., total use of inputs, total production, economic accounts and total emissions) for each farm type are being calculated. Such approach provides insight into responses at farm level to policy measures imposed through scenarios.

Having estimated the impact of policies for each of the selected farm types allows to undertake further analysis regarding structural changes of the overall agricultural sector and its impact on climate change through GHG emissions.

16. Farm module database covers primary production factors, inputs and outputs. Data are expressed in physical units and converted to monetary terms using estimated price level. Thus, the consistency is ensured between quantities, values and prices. Data are sourced from the harmonised datasets of GUS and Polish FADN[4], as well as verified on the basis of publicly available data of regional Polish Agricultural Advisory Centres[5] and several other sources (more in Section 2.4).

## 2.2. Module structure

17. In order to comprehensively assess responses of agricultural sector to policy measures the farm module (Figure 3) at its basic level is divided into interlinked crop and animal production, each represented by both extensive and intensive production intensities (referred to as *technology*) and contribution to land use change. As the farm income is derived from total revenues subtracted by total costs, the farm revenues are calculated based on farm gate prices of crop and animal commodities multiplied by either yield per hectare based on the devoted area (for crop production) or production output per LU in regard to current number of animals (for animal production). These are estimated for both extensive and intensive types of production. On the farm costs side: 1) crop production is defined by the fertiliser inputs (purchased mineral and own natural fertilisers, according to modelling assumptions), crop residuals and other inputs (seeds, planting materials, pesticides and other), while 2) animal production costs include feed (purchased concentrate and own roughage, according to modelling assumptions), other inputs (veterinary services, medicines, insemination, milk yield control, etc.). The model may also include payments for generated GHG emissions and subsidies for avoided

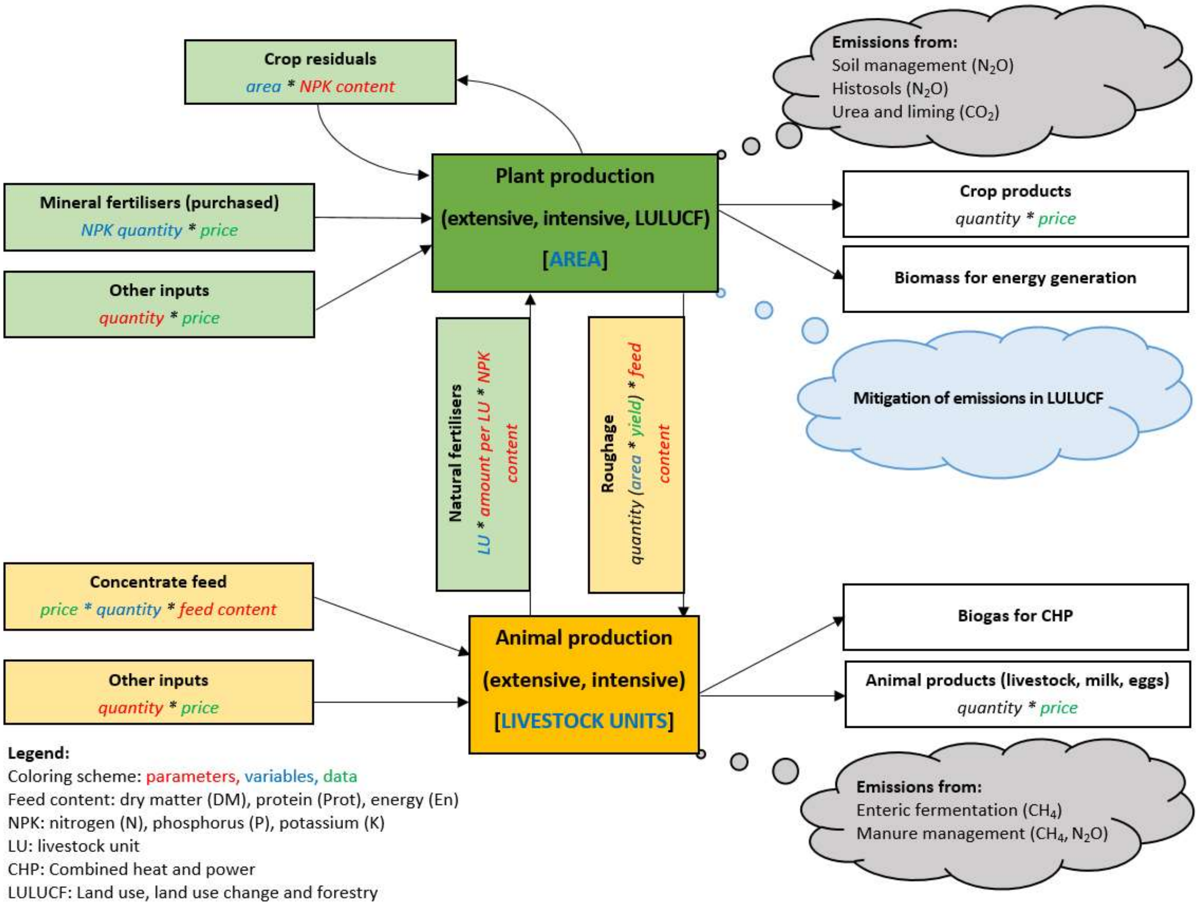
[4] FADN is a Farm Accountancy Data Network, being an instrument for evaluating the income of national agricultural holdings and the impacts of the Common Agricultural Policy. Each EU Member State collects relevant farm information based on selected sample. While there were overall 1,506,620 farms in Poland in the 2015, only the farms with standard output (SO) of 4,000 EUR and over were considered by the FADN. Therefore the number of such farms accounted to 730,879, which were responsible for 93.03% of the total production value in agricultural sector, cultivated 85.07% of agricultural land, kept 96.9% of farm animals, while employing 66.46% of agricultural labour resources (Polish FADN 2013). Representing these farms a total of 12,100 farms have been selected as the Polish FADN sample, data of which serves as the basis for assessment of Polish agriculture in the farm module of the EPICA model. Those farms cultivate 88% of agricultural area and produce 98% of farm animals.

[5] Agricultural Advisory Centre (including central and regional offices) is a government institution subordinated to the Ministry of Agriculture and Rural Development of Poland. The purpose is the improvement of knowledge and qualifications of advisory staff as well as increase and unification of standards of services provided by advisers for farmers and other rural dwellers. Source: https://www.cdr.gov.pl.

GHG emissions. On top of revenues and costs the available payments within CAP are taken into account to ensure the most comprehensive picture of farming.

18. Produced commodities are divided into primary crop and animal outputs, which go along the relevant GHG emissions. GHG emissions as one of the key estimation targets in the EPICA model are evaluated based on each farm activity output, for crop production as $CO_2$eq/ha and for animal production as $CO_2$eq/LU. Emissions from crop production cover such sources as: 1) soil management ($N_2O$), 2) histosols ($N_2O$), and 3) urea and liming ($CO_2$). Emissions from animal production cover: 1) enteric fermentation ($CH_4$), and 2) manure management ($CH_4$, $N_2O$).

**Figure 3. EPICA's farm module detailed input-output concept**




Source: *CAKE/KOBiZE own study*

19. The EPICA model goes deep into assessment of specific agricultural activities. There are in total 23 activities singled out, distributed between 17 crop and 6 animal farm activities, having their distinctive input-output assumptions with consideration of two types of production intensity (extensive, intensive) with the final output of primary products and accompanying GHG emissions. These activities are performed by farms in agricultural sector, the total of which are aggregated into 19 types, according to their

specialisation and size criteria. Each farm in the base year is calibrated to match observed structure of farm activities. The structure of activities in each of the farm types is subject of optimisation procedure.

20. Apart from typical agricultural crops the model includes GHG mitigation measures available through crop-based activities as afforestation of agricultural land and restoration of wetlands on histosols. Production of biomass for energy purposes is also considered as marketable crop production, which causes emissions in agricultural sector, but provides renewable fuel supply for the energy sector.

21. To ensure flawless exchange of data and comparability between parts of the model several assumptions have been made in regard to units used in the model. Thus crop nutrients are reflected by the chemical elements of nitrogen (N), phosphorus (P) and potassium (K), which aside of being purchased in the form of mineral fertilisers, are also supplied to crops in form of natural fertilisers, crop residuals and other natural sources. For the animal nutrients the assumed elements include the dry matter (DM), net energy lactation (En) and crude protein (Prot), supplied by the production of fodder crops and purchase of feed concentrates.

## 2.3. Key terms

22. The following section is devoted to key terms used within the farm module, with distinctive definitions of general terms, as well as sector-specific crop and animal production terms.

### A. General terms.

23. **Farm type** – all the farms in agricultural sector are aggregated into 19 types (Annex, Table 2). Six agricultural specialisations include: 1) cereals, 2) crops (all excluding cereals), 3) cattle, 4) pigs, 5) mixed and 6) other. Each of these 6 types are split according to their size into small, medium, and large, resulting in overall of 18 types. There is an additional farm type defined in order to represent the rest of the agricultural sector, namely the semi-subsistence farms, which are not considered as producers of marketable products, yet represent nearly half of the overall farm population in physical terms.

24. **Farm activity** – type(s) of agricultural production carried out by particular farm type (Annex, Table 2), with a total of 23 farm activities outlined. Crop production activities include 17 groups and 2 LULUCF activities, while the animal production – into 6. The outlined crop activities could be grouped according to the purpose of these products. These would be: 1) general crops (including wheat, other cereals, oilseeds, sugar beets, potatoes, proteins (for grain), maize (for grain), fruits (short term <5 years),

vegetables (short term <5 years), fruits & vegetables (>5 years); 2) fodder crops (including proteins (for fodder), maize (for silage), permanent grassland, grass on arable land and other fodder crops; 3) energy crops; 4) other crops; and 5) fallow land (Ecological Focus Area). Animal activities are distributed into the following: cattle for beef, dairy cattle, pigs for meat, poultry for meat, poultry for eggs, other animals.

25. **Prices** – the ones used in the model refer to the farm gate prices. For each farm the farm gate price is derived from sales values and volumes of produce sold.

26. **Production intensity (technology)** – all of the farm activities are presented in the module as both intensive and extensive, which are derived from the character of production processes and linked to the yields (output). While technologies generally differ between crop and animal production, they are also deeply specific for each farm activity in regards to implemented or potential farming practices. There are several crop farm activities which do not differ by the production intensity due to typical type of the land use (permanent grassland, fallow land).

27. **Inputs** – production inputs vary by the farm activities and are also subject to change based on the implemented technology. Inputs are also differed by own and purchased, as in case of mixed types of farms a possibility exists of conducting complimentary farm activities, which cover parts or whole production chain (e.g., cattle farming and grasslands).

28. **Nutrients** – substances used to produce both crop and animal biomass; inputs differ depending on crop and animal production, as well as differ according to their source - they are purchased or produced by the farm itself.

29. **Emissions** – three greenhouse gases are covered by the module, including the nitrous oxide ($N_2O$), methane ($CH_4$) and carbon dioxide ($CO_2$). To ensure consistency of the analysis they are presented by the $CO_2$ equivalent ($CO_2$eq) based on the GHG Global Warming Potential values from the IPCC Fourth Assessment Report (Forster et al. 2007) for the 100-year time horizon. These values equal: for the $N_2O$ – 298, for the $CH_4$ – 25, and for the $CO_2$ – 1.

### B. Crop production terms.

30. **Crop activities** are represented in the module as 17 farm activities, most of which are revealed in two production technologies. Intensive technology of crop production is characterised by high level of inputs and corresponding high yields, while the extensive ones are designed to capture low input agricultural practices. In each type of farming the mix of intensive and extensive version is used for each crop activity to represent intensity level of production characteristics for considered farm types. Possibility of changing proportion between intensive and extensive activities in optimisation process

allows to observe impact of scenarios on intensity of production (level of inputs per hectare of land). Each crop production activity is described by the following set of parameters: yield (t/ha), farm gate price (CU/t), nutrient requirements (N, P, K, in kg/ha), nutrients leftovers (N, P, K, in kg/ha), emissions generated ($CO_2$eq/ha), and other costs. These parameters are introduced to the model as common (fixed for all farm types), yet separately for the extensive and intensive crop production activities. The variability of production systems and practices between the farm types is reflected by the shares of the intensive and extensive activities. Thus, the actual value of each parameter in each of the farm type depends on proportion of intensive and extensive practices, which for the baseline year is estimated based on the yield observed in each of the farm types.

31. **Yields for crop activities** – yields are an important parameter used in the EPICA model defined as the volume of production of the main commodity harvested on the given area[6]. Yields for crop activities in the baseline year are estimated using FADN data on production and area used for particular activities independently for each farm type modelled. The yields are estimated based on produced quantities and the area of land used by each farm activity. For activities representing the number of different crops (e.g., fruits and vegetables) the yield is estimated as the value of produced crops, otherwise physical values are used (tonnes/ha).

32. **Crop nutrients** – are the factors affecting crops' growth rate controlled by farmers through the level of nutrients in the soil. In our analysis the main soil nutrients considered are nitrogen (N), phosphorus (P) and potassium (K). Crops are using nutrients for production of biomass. Only part of the biomass is harvested as a main crop (e.g., cereals grain), while the rest is harvested as a supplementary crop to remain on the fields as residuals (e.g., straw). Thus the uptake of nutrients per unit of harvested yield is not fixed as it depends on the ratio of the main crop (e.g., grain) to by-products (e.g., straw) and other parts of the plant (e.g., roots). Typically the uptake of nutrients depends on the crop growth as each of the crops has different characteristics and the amount of the biomass produced. Due to that the ratio of nutrient uptake to the yield depends on the level of yield and therefore – the intensity of production. In general there are four sources of crop (soil) nutrients i) natural sources (e.g., nitrogen fixation form air), ii) crop leftovers, iii) natural fertilisers (e.g., animal manure), and iv) mineral (purchased) fertilisers. Nutrients from natural sources depend on natural conditions and are usually taken as fixed for the whole area. Amount of nutrients from natural sources is assumed as fixed for each hectare throughout the country.

33. **Crop leftovers** – unharvested biomass (e.g., straw of cereals, leaves of potatoes) remaining in the soil being the source of nutrients for next crop in the next year. The amount of leftovers is estimated based on the literature assuming typical practices used within intensive and extensive activities (Grześkowiak 2016, Igras 2013). As

[6] One hectare of land is used as a reference unit in Europe.

amount of the leftovers is strongly differentiated between crops the level of available nutrients in the following year depends on the crop structure and cultivation intensity. Therefore in the model the amount of nutrients originating from crop residuals is calculated endogenously as it depends on the production structure patterns (e.g., structure of crops and production intensity).

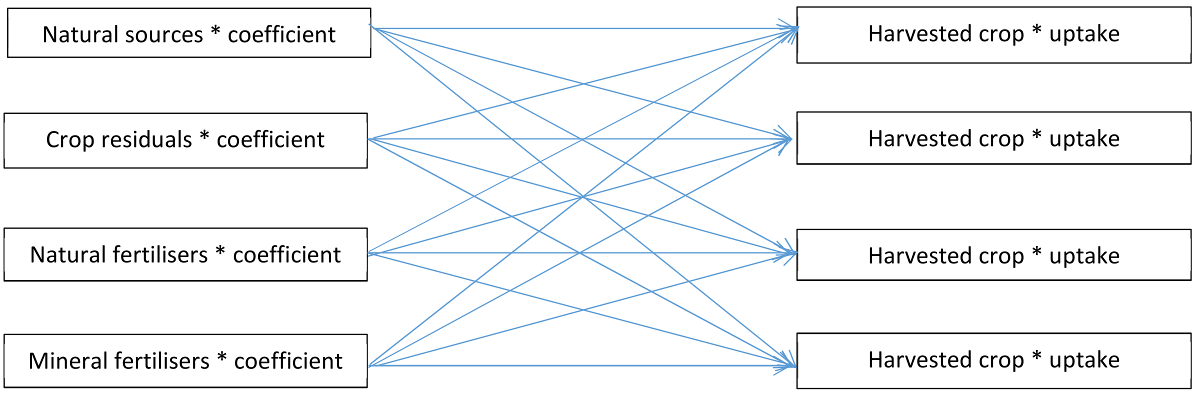
34. **Natural fertilisers** are by-products of animal production process. In case there is a mixed farm with both animal and crop production, it could be a significant source of its own crop nutrients. Yet even if the farm cannot provide the required volumes of natural fertilisers (in addition to natural sources and crop leftovers) to comply with the needed nutrients uptake of the cultivated crops' these needs can be supplemented by the mineral fertilisers. Nowadays it is a typical practice in agriculture and most farmers are using fertilisers to level the nutrients' balance and keep yields on the desired level. Magnitudes of nutrients from the natural fertilisers are also calculated as endogenous and they depend on number of animals kept on the farm. Additional constraint is included to ensure modification of distribution of nutrients from manure in case of pastures. It needs to be taken into account since part of the manure produced by the grazing animals is remaining on the pastures and could not be distributed/moved to other crops.

35. **Mineral fertilisers** – model assumption is that this source of crop nutrient is unlimited to the farmer, yet compared to other three sources has to be purchased if needed. Therefore the use of nutrients from mineral fertilisers is associated with additional costs (calculated as fertiliser's price times the amount of purchased nutrient). Mineral fertilisers available on the market typically contain only proportion of needed nutrients (e.g., ammonium nitrate contains 34% N, superphosphate 46% $P_2O_5$ equalling nearly 20% pure P, sylvinite 60% $K_20$ ≈ 50% pure K). Due to these peculiarities for the model purposes the price of NPK nutrients is calculated based on prices of most popular (widely-used) fertilisers on the domestic market. Purchase of nutrients is a decision variable in the model and it is endogenous, thus giving potential to capture mitigation of emissions resulting from application of mineral fertilisers.

36. **Nutrients' balance** – as the nutrients could be acquired from different sources the nutrients balances are designed to ensure proper amount of required nutrients within the farm (in regard to its activities). It is assumed in the model that amount of nutrients used for crop production has to be covered from available nutrient sources. To construct balance of nutrients for the farm module (Figure 4), the characteristics of each crop activity has to include coefficients of nutrient uptake (amounts of nutrients needed by the crops in relation to expected yield level) and amount of nutrient included in the crop leftovers. Except mineral fertilisers (which need to be purchased according to model assumptions) the other sources of nutrients are directly connected with farming activities and can be used with no additional costs. Thus the rational nutrient

management will aim at using resources which are already available at the farm and only the missing part will be delivered with purchased mineral fertilisers. It also needs to be taken into account that not all nutrients delivered from above mentioned sources are immediately available for plants. The proportion of nutrients available from each source is dependent on biophysical processes within the soil in which nutrients are converted into the chemical form which could be acquired by plants. Thus each of the source is characterised by coefficient describing nutrients availability (e.g., only 35-50% of nitrogen from the solid manure is available in the first year after its application - IUNG 2019).

**Figure 4. Scheme of nutrients' balance in crop production**




Source: *CAKE/KOBiZE own study*

37. **Fodder (animal feed)** – is typically acquired by the farm from different sources (crop production, purchase) and depends on the specific type of animal. Cattle is being fed with roughage (assumed in the model to be produced by the farm itself in case of mixed farms) and concentrate feed (assumed to be purchased). However, due to the characteristics of the animal activities, it is also assumed that pigs and poultry are not fed by roughage, while in case of cattle the appropriate proportion between roughage and concentrate feed is ensured by dry matter balance. Even though the purchase of the animal feed is a decision variable it is needed to mention that every animal feed available on the market consist of given combination of dry matter, energy and protein. To differentiate composition of purchased feed it was assumed to introduce two decision variables describing feed purchase: low protein concentrates (represented by cereals' grain) and high protein concentrates (represented by soya cake). The roughage is produced by growing grass (either permanent or temporary), maize for silage, proteins for fodder or even on grass on fallow land. Concentrate feed is produced from cereals, processed crops (soya cake, rapeseed cake) or other industrial sources.

Key model assumption in terms of animal feed states that all the roughage is produced by the farm itself, yet all of concentrated feed is being purchased on the market. In case part of the crops produced on the farms is directly used as feed it is assumed in the model that these cereals or proteins (for grain) are first sold to the market and later purchased in desired quantities (if needed).

38. **Animal feed balance** – represents sufficient amount of feed for all animal activities on the farm. The model uses a relatively simple description of the farm feeding system based only on three key elements: dry matter (DM), net energy lactation (NEL) and crude protein (Prot). It is assumed every animal activity should be provided sufficient amount of energy and proteins, as well as a reasonable amount of dry matter (e.g., it is not feasible to cover energetic and protein needs of the cow with solely the cereal straw, as the cow would not able to consume the amount needed to reach protein and energy balance; it is also not possible to cover cow's needs with solely concentrated feed, as it would not be satisfactory in terms of intake volume, in which dry matter helps). The amount of dry matter per one animal is restricted by a threshold. Overall, similarly as in case of nutrients in crop production the animal feed is distributed among the animal activities.

39. **Other costs in crop production** – apart of microelements included in nutrients balance the crops require other inputs. These are mainly seeds or other planting materials (e.g., seedlings, seed potatoes), pesticides and microelements. To simplify the model such inputs are introduced only in monetary terms. Other costs include farmers' expenditures calculated per hectare of crop activities in regard of all inputs apart the crop nutrients, labour and fuel necessary to cultivate considered crops. The parameters are estimated for both intensive and extensive production separately based on the reports of agricultural advisory services (MODR 2020). Other costs are calculated for every farm type based on the achieved yields. In case value of these costs adjusted to the yield for all crops in particular farm type is different than those recorded in the FADN database an appropriate correction coefficient is applied to the farm type to match the FADN records.

40. **Labour in crop production** – for each of the crop activities the necessary labour inputs required to cultivate particular crops are defined. Based on the normative data the amount of labour needed to grow each of the crops in intensive and extensive technology varies. Additionally, based on the compiled literature data (CDR Brwinów 2020) and FADN records for each of the farm types, a coefficient is used describing labour efficiency (as cultivation of the same crops in larger farms usually requires less inputs due to relatively better equipment).

41. **Fuel cost in crop production** – most of the farm machinery in crop production is powered by the diesel engines. These engines are installed mostly in tractors or combine harvesters. Thus, the machinery inputs are determined by the amount of fuel needed to

perform necessary actions. Depending on the size of machinery and its efficiency a certain amount of fuel is required to perform all operations related to cultivation of particular crops. For each of crop types in both technologies (intensive and extensive) the amount of fuel used by machinery is estimated based on normative data. An efficiency coefficient for each farm type is then applied, which helps to reflect their particular efficiency.

42. **Emissions from crop production** – each of the crop activities is characterised by a specific combination and level of GHG emissions. For both intensities of each crop production processes the coefficients defining the emissions of GHG are estimated in relation to the yields counted at the farm gate level. The emissions characterising each activity in every of the considered farm types are derived from a mix of intensive and extensive versions of crops.

### C. Animal production terms.

43. **Animal activities** – represented in the module by 6 farm activities: cattle for beef, dairy cattle, pigs for meat, poultry for meat, poultry for eggs, and other animals. The scale of the animal production activities is measured in terms of livestock units[7] in order to capture the diversity of the animals kept on the farm and make the comparison possible. The last group "other animals" is very diverse, as it consists of different animals of marginal importance for the animal production sector and should be treated as residual animal activity. It has to be noted that it is not possible to precisely predict potential changes in the structure within "other animals" group, therefore it was treated in the model as rather the one to maintain the balances of animal production activities.

44. **Yields in animal activities** – in case of animal activities the yield is the production output collected per LU on the farm (e.g., volume of milk, number of eggs, or number of live animals sold for further processing). Large differences in the yield, measured in terms of output per one LU, exist depending on the production intensity, as if the animals are kept in more intensive production system the fattening period is shorter and the production per LU is greater. In case more than one commodity is produced on a farm due to animal production activity (e.g., both milk and beef in case of dairy cows), the yield of both commodities is estimated for these activities.

45. **Animal feed requirements** – for each of the modelled animal activities feed requirements are specified by a fixed coefficient. Animal feed requirements are

[7] The livestock unit [LSU or LU] is a reference unit which facilitates the aggregation of livestock from various species and age as per convention, via the use of specific coefficients established initially on the basis of the nutritional or feed requirement of each type of animal. The reference unit used for the calculation of livestock units (=1 LSU) is the grazing equivalent of one adult dairy cow producing 3,000 kg of milk annually, without additional concentrated foodstuffs (Eurostat 2013).

measured as energy intake expressed as amount of: i) dry matter [kg/LU/year], ii) amount of energy [MJ/LU/year], and iii) crude protein [kg/LU/year]. This coefficient describes how much of feed is needed to maintain certain production level. Similarly as in case of yields the feed requirements are estimated for both extensive and intensive production systems.

46. **Animal manure** – for most animal activities one of the key by-products is the animal manure, which is used as a natural fertiliser. Depending on the production system there are different forms (e.g., solid manure, liquid manure, slurry) in which the animal manure could be produced. Solid manure output predominates in traditional (being rather extensive) cattle and pig breeding systems, while in more intensive systems a liquid manure (slurry) is more likely to be produced. In the poultry farms only solid manure is produced. Solid and liquid manures differ in regard to daily amounts produced and concentration of nutrients (N, P, K) per animal/LU. The proportion of liquid and solid manure in intensive and extensive animal activities was assumed for each animal group separately using literature data (KOBiZE 2015).

47. **Other costs in animal production** – other operational costs to be covered by the farmer to maintain production are the veterinary services, medicines, insemination, milk yield control and other. These costs are reflected only in monetary means (currency units/LU/year) and are estimated based on FADN data of farms specialising in particular animal production activities and verified by calculations published by regional Polish Agricultural Advisory Centres (CDR).

48. **Labour in animal production** – for each of the animal production activities the necessary labour inputs required to maintain the particular animal activity are reflected in the model. Based on the literature data (CDR Brwinów 2020) the amount of labour needed to upkeep each of the animals in intensive and extensive type of production is defined in the model. Additionally, based on the normative data and FADN records for each of the farm types a coefficient describing labour efficiency is being used, as the same animals in farms with higher production scale usually require less inputs due to better infrastructure (e.g., buildings, equipment).

49. **Fuel in animal production** – most of the animal production equipment is powered by electric engines, however transportation of roughage (feed) in case of cattle might require the use of diesel powered machinery (tractor), thus amount of fuel used for these activities is assumed based on the FADN data for every farm type and scale of production. It is reflected in litres of diesel per LU per year.

50. **Emissions from animal production** – in case of animal activities the GHG emissions consists mostly of methane and are caused by enteric fermentation and manure management. Amount of GHG emitted due to enteric fermentation is dependent mostly on the animal activity and production intensity, however, it might also be influenced by

the feed composition. The methodology for calculation of emissions caused by enteric fermentation is given by the IPCC (2006). For each of the animal activities the emission is calculated per LU/year. Emission of the GHG due to manure management system needs has been assumed individually for each of the farm type. The emission from manure depends on the infrastructure, type of manure managed on the farm and the period these natural fertilisers' storage. An assumptions are made on the dominating type of animals on the farm, type of the dominating buildings for the animals' upkeep, as well as the scales of animal activities in relation to the area of arable land.

**D. GHG mitigation measures.**

51. **LULUCF (Land use, land use change and forestry) activities** are represented in the farm module by: 1) afforestation of agricultural land, and 2) restoration of wetlands on histosols, which combined are responsible for 8% of GHG emissions in agricultural sector. It is assumed that forest planted on agricultural land would sequestrate $CO_2$ at the level of an average existing forest in Poland. Recovering wetlands on the histosols would reduce $N_2O$ emissions (IPCC Agriculture) while decreasing $CO_2$ emissions (IPCC LULUCF).

52. **Energy crops** are considered in the model as a typical crop production. However, one of the effects of this activity is generating GHG emission in the agricultural sector (e.g., due to application of nitrogen fertilisers), yet providing a renewable fuel supply for the energy sector.

53. **Agro-biogas** activity is presented through processing of manure produced by farm animals. This activity requires significant investments, which are included in the model in form of depreciation. In the process of biogas production, along with the generated energy, the emissions from manure management are being reduced. Digestate from biogas plants is to be delivered to the fields, thus the nutrients' balance is not affected.

## 2.4. Data

54. Structural economic models typically represent a snapshot of the sector in a given year. For that reason the data used in the EPICA's farm module characterises the situation at analysed farms in a given moment of time. For calibrating the baseyear the data from 2015 year were used. The data used in the farm module could be generally divided into three groups. The first set of data are the characteristics of modelled farm types (e.g., available resources (land, animal stalls or labour), cropping structure in a given year, productivity level), the second set of data are the characteristics of modelled farm activities (e.g., yield of particular crop activity or price of the commodity produced), and

the third set is the general data describing interrelationships between the considered farm activities (e.g., crop nutrients or animal feed balances).

55. Data are typically expressed in both physical and monetary terms, in case of some data (see Section 2.3) they are reflected only in monetary terms. Mutual consistency is ensured between quantities, values and prices. Data are sourced from the harmonised datasets of GUS, Polish FADN and KOBiZE, as well as verified on the basis of publicly available data of CDR and regional Polish Farm Advisory Centres (ODR).

56. Data regarding agricultural production in Poland was derived from the publications of Statistics Poland. It was aggregated in order to ensure consistency between the FADN sample data and the national statistics, as well as to verify the module assumptions. Data used includes the utilised agricultural land area, area under particular crops, production volumes and values of crops, quantity of farm animals, animal production in terms of quantity and value. Data derived from this sources was aggregated according to the structure of farm activities set in the farm module.

57. Another key source of data is FADN - Farm Accountancy Data Network, being an instrument for evaluating the income of national agricultural holdings and the impacts of the Common Agricultural Policy. In regard to crop activities, the Polish FADN sample records a total of ca. 150 various crops. Production of many of crops has however only marginal meaning and for the sake of simplicity could be grouped into aggregates without any implications on the results of our analysis. In order to capture response of production structure in analysed farm type keeping reasonable size of the model and ensuring linking capability with more aggregated core d-PLACE model, the set of crops has been delimited.

58. Regional branches of Agricultural Advisory Centre in Poland (MODR 2020) provide detailed calculations for various types of agricultural production. This data includes detailed description and calculation of production inputs and outputs, including the monetary and physical values. This data has served as a verification basis for farm economic accounts (prices, yields, costs, nutrient inputs, payments) for both crop and animal production.

59. The baseline in the EPICA's farm module is built on 2015 data.

## 2.5. Key equations

60. **Objective function** of farm module is the maximisation of farm income (Annual farm income of each farm type = Revenues – Costs (except own land and own labour) + Payments):

$$PROF = \sum_{act}\sum_{tech}\Big(\big(price_{(act,tech)} * yield_{(act,tech)} + paymt_{(act,tech)} - ocost_{(act,tech)} + taxsub_{(act,tech)}\big) * XACT_{(act,tech)}\Big) - \sum_{month}(HRLAB_{month} * wage) - \sum_{pinput}\big(PURC_{pinput} * iprice_{pinput}\big) + \sum_{fixp} farm_{dat(fixp)} - \sum_{fixc} farm_{da\ (fixc)} - GHG_{emi} * ETS_{price} + GHG_{avoid} * ETS_{sub} - \sum_{ghgmit}\big(YCOST_{ghgmit} * AREAT_{ghgmit}\big) - (XBIO_{netinc} * XBIO),$$

where:

act = farm activities,
tech = production technologies (intensities),
$price_{(act,tech)}$ = farm gate prices per unit of activity [PLN/ton],
$yield_{(act,tech)}$ = yield units of commodities per activity unit (tonnes per ha or tonnes per LU),
$paymt_{(act,tech)}$ = activity related payments PLN per unit of activity (ha or LU),
$ocost_{(act,tech)}$ = other direct costs (costs not directly related to GHG emission),
$taxsub_{(act,tech)}$ = balance of taxes (-) and subsidies (+) dependent on structure of farm activities – to be used in considered climate policy scenarios [PLN],
$XACT_{(act,tech)}$ = size of farm activities [ha, LU] – decision variable,
$HRLAB_{(month)}$ = hired labour [hours] – decision variable,
$PURC_{pinput}$ = amount of purchased inputs [dt] – decision variable,
$iprice_{pinput}$ = farm gate price per unit of input [PLN/dt],
farm_dat (fixp) = farm type specific data - fixed payments [PLN],
farm_dat (fixc) = farm type specific data - fixed costs [PLN].
$GHG_{emi}$ = GHG emissions calculated based on levels of activities according to farm types.
$ETS_{price}$ = price of GHG emission allowance for agricultural sector.
$GHG_{avoid}$ = GHG emissions avoided according to farm types.
$AREAT_{ghgmit}$ = size of area-based GHG mitigation activities [ha].
$YCOST_{ghgmit}$ = annual cost (capex+opex) per one hectare of crop production mitigation activities [PLN/ha].
$XBIO_{netinc}$ = net income per one kWhe produced from agro-biogas.
XBIO = size of biogas production activities in kWhe.

However in the optimisation process several constraints must be specified to result in a solution. In case of farm models those solutions usually describe utilisation of farm resources, either produced or purchased. Below the main farm module constraints (balances) are presented.

61. **Land balance (constraint)**. Land is the production factor resources of which cannot be increased on a sector scale. Even though it is possible to increase the size of the farm it was assumed that land resources of all considered farm types remain at the baseline level. Such assumption allows to control the amount of the land used within the country. The balance is included in the model in a form of equation:

$$\sum_{crop_{(act)}} \sum_{tech} XACT_{(act,tech)} + \sum AREAT_{ghgmit} \leq farm\_dat_{(UAA)},$$

where:

$crop_{(act)}$ – crop activities;
tech - production technologies (intensities);
$XACT_{(act,tech)}$ – size of farm activities [ha, LU] – decision variable (see par. 55);
$farm\_dat_{(UAA)}$ - farm data in regard to utilised agricultural area [ha].

62. **Stable stalls balance (constraint).** Similarly the existing premises for animals have been reflected as the maximum stable stalls number, which is assumed to be fixed in the baseline year in each of the farms. However it was also assumed that the stable stalls could be adjusted and the place spare due to decrease in one group of animals could be used by any other animals. This balance is presented in the equitation below:

$$\sum_{head_{(act)}} \sum_{tech} XACT_{(act,tech)} \leq farm\_dat_{(stable)},$$

where:

$head_{(act)}$ = animal activities,
tech - production technologies (intensities),
$XACT_{(act,tech)}$ = size of farm activities [ha, LU] – decision variable (see par. 55),
$farm\_dat_{(stable)}$ = farm data in regard to stable stalls.

63. **Labour balance (constraint).** The labour resources are the factor that is constrained in the farm module. The labour resources at farm level are divided into two groups. The labour of the farmer and family is a resource that could be used free of additional charge, thus generating no costs, as all the farm income is received by the farmer and his family. However if internal labour resources are not sufficient to perform production the farm can employ workers, what induce additional costs. The number of workers

employed is a model variable. The overall labour balance is presented through the equation:

$$\sum_{crop(act)} \sum_{tech}(labreq_{(act,tech,month)} * XACT_{(act,tech)}) + \sum_{head(act)} \sum_{tech}(labreq_{(act,tech,month)} * XACT_{(act,tech)} \leq \left(farm\_dat_{(LABR)} * normres_{(month)}\right) + HRLAB_{(month)},$$

where:

act - farm activities,

tech - production technologies (intensities),

$crop_{(act)}$ - crop activities,

$head_{(act)}$ - animal activities,

$labreq_{(crop,tech,month)}$ = labour requirements per unit of activity per month [hours],

$XACT_{(act,tech)}$ = size of farm activities [ha, LU] – decision variable (see par. 55),

$farm\_dat_{(LABR)}$ = farm data in regard to own (unpaid) labour resources [hours],

$normres_{(month)}$ = number of working hours per month per one person,

$HRLAB_{(month)}$ = monthly hired labour – [ha, LU] decision variable (see par. 55).

Due to seasonality of labour the amount of labour is balanced on the monthly bases.

64. **Crop nutrients' balances (constraint).** The circulation of biogenic elements on the farm was designed to ensure covering of demand for nutrients due to crop production by delivering nutrients produced on farm itself (natural sources, produced manure, crop residuals) or purchased as fertilisers. This balance is described by the following equation:

$$\sum_{crop(act)} \sum_{tech}\left(cropres_{(crop,tech,fert)} * XACT_{(act,tech)}\right) + \sum_{head(act)} \sum_{tech}\left(manu\_cont_{(head,tech,fert)} * XACT_{(act,tech)}\right) + \left(farm\_dat_{(UAA)} * nat\_src_{(fert)}\right) + \sum_{pfert(pinput)}\left(pfert\_cont\_av_{(pfert,fert)} * PURC_{(pinput)}\right) \geq \sum_{crop(act)} \sum_{tech}\left(crop\_uptake_{(crop,tech,fert)} * XACT_{(crop,tech)}\right),$$

where:

act = farm activities,

tech = production technologies (intensities),

$cropres_{(crop,tech,fert)}$ = nutrient residuals (N,P,K) by crop per unit of yield in given technology [kg/ha],

$XACT_{(act,tech)}$ = size of farm activities, [ha] decision variable (see par. 55)

$manu\_cont_{(head,tech,fert)}$ = crop nutrients amount (N,P,K) in animal manure [kg/LU],

$farm\_dat_{(UAA)}$ = farm data in regard to utilised agricultural area [ha],

$nat\_src_{(fert)}$ = nutrients (nitrogen) from natural sources per ha of UAA [kg N],

$pfert\_cont\_av_{(pfert,fert)}$ = crop nutrients content in purchased fertilisers [kg/dt],

$PURC_{pinput}$ = amount of purchased inputs [dt] – decision variable (see par. 55),

$crop\_uptake_{(crop,tech,fert)}$ = uptake of nutrients (N,P,K) by crop per unit of yield in given technology [kg/dt].

It has been assumed that the crop nutrients (Nitrogen, Potassium and Phosphorus) necessary for the optimal level of production can be supplied from four sources: three internal (natural sources, crop leftovers, animal manure) and one external (purchased). Therefore the balances and nutrient needs are based on the following assumptions:

1) **Nutrients from the farm** + purchased nutrients – crop requirements ≥ 0.
2) **Nutrients from the farm** = natural sources * area * AC + nutrients from crop leftovers * crop area * AC + manure from animals * animal number * nutrient content * AC,

where:

$AC_{(nutrient\ source)}$ = availability coefficient = proportion of nutrients from each source available to crops.

3) **External nutrients** = purchased nutrients * AC.
4) **Crop requirements** = yield * nutrient uptake * area.

65. **Animal feed balances (constraint).** Similarly animal feed requirements were described. However, due to varying feeding requirements and techniques in regard to ruminants and granivores, the feeding balances consist of four different equations. The feed requirements were considered in regard to three crucial elements: dry matter, crude proteins and metabolic energy.

In case of ruminants it was assumed that dry matter, crude proteins and energy should be covered by production of own fodder crops (roughage harvested from crop activities grown strictly on the farm) and from purchased concentrated feed (e.g., soya cake or cereals).

The balance used in the model is represented by the equation:

$$\sum_{pfe\ (pinput)} \left(pfeed_{cont(pfeed,nutr)} * PURC_{(pinput)}\right) + \sum_{fodr(crop)} \sum_{tech} \left(yield_{(crop,tech)} * crop_{cont(crop,tech,nutr)} * XACT_{(crop,tech)}\right) \geq \sum_{rumi(head)} \sum_{tech} \left(head\_req_{(head,tech,nutr)} * XACT_{(head,tech)}\right),$$

where:

$pfeed_{(pinput)}$ - purchased roughage feed,
$pfeed\_cont_{(pfeed,nutr)}$ = content of animal nutrients in purchased concentrated feed [kg/dt, |MJ/dt],
$PURC_{pinput}$ = amount of purchased inputs [dt] – decision variable (see par. 55),
$fodr_{(crop)}$ = fodder crops,
$yield_{(crop,tech)}$ = yield per crop activity unit [dt/ha],
$crop\_cont_{(crop,tech,nutr)}$ = content of animal nutrients in fodder crops [kg/dt, MJ/dt],
$XACT_{(act,tech)}$ = size of farm activities [ha, LU] – decision variable (see par. 55)
$rumi_{(head)}$ = number of ruminant animals – decision variable – subset of XACT [LU],

$head\_req_{(head,tech,nutr)}$ = feed requirements of animal nutrients per animal in given technology [kg/LU, MJ/LU].

Additionally maximum intake of dry matter for ruminants has been constrained to avoid use of cheap low value fodder in large quantities, which could exceed maximum physical intake of farm animals. It was assumed that maximum intake of dry matter could be 15% higher of the minimum requirement, which is presented in equation:

$$\sum_{pfeed_{(pinput)}}\left(pfeed\_cont_{(pfeed,DM)} * PURC_{(pinput)}\right) + \sum_{fodr_{(crop)}} \sum_{tech}\left(yield_{(crop,tech)} * crop\_cont_{(crop,tech,DM)} * XACT_{(crop,tech)}\right) \leq \sum_{rumi_{(head)}} \sum_{tech}\left(head_{req_{(head,tech,DM)}} * XACT_{(head,tech)}\right) * 1.15,$$

where:

$pfeed_{(pinput)}$ - purchased roughage feed,
$pfeed\_cont_{(pfeed,DM)}$ = content of dry matter in purchased fodder [kg/dt],
$PURC_{pinput}$ = amount of purchased inputs [dt] – decision variable (see par. 55),
$fodr_{(crop)}$ - fodder crops,
$yield_{(crop,tech)}$ = yield per crop activity unit [dt/ha],
$crop\_cont_{(crop,tech,DM)}$ = content of dry matter in fodder crops,
$XACT_{(act,tech)}$ = size of farm activities [ha, LU] – decision variable (see par. 55),
$rumi_{(head)}$ = number of ruminant animals [LU],
$head\_req_{(head,tech,DM)}$ = feed requirements of dry matter per animal in given technology [kg/LU/year].

Similar approach has been used for granivores (pig and poultry), however it was assumed that the animals are fed only with the purchased feed. However, the variety of possible feed mixtures is more diversified here, including cereals, soya cake, pig feed mix, meat chicken feed mix, laying hens feed mix. The balance is expressed by the equation:

$$\sum_{pconc_{(pinput)}}\left(pfeed_{con\ (pconc,nutr)} * PURC_{(pinput)}\right) \geq \sum_{gran_{(head)}} \sum_{tech}\left(head\_req_{(head,tech,nutr)} * XACT_{(head,tech)}\right),$$

where:

$pconc_{(pinput)}$ - purchased concentrate feed,
$pfeed\_cont_{(pconc,nutr)}$ = content of animal nutrients in purchased concentrate feed, [kg/dr, MJ/dt]
$PURC_{pinput}$ = amount of purchased inputs [dt] – decision variable (see par. 55),
$gran_{(head)}$ = number of granivore animals decision variable – subset of XACT [LU],
$head\_req_{(head,tech,nutr)}$ = feed requirements of animal nutrients per animal in given technology, [kg/LU, MJ/LU].
$XACT_{(act,tech)}$ = size of farm activities [LU] decision variable (see par. 55).

Even though the probability of excessing the dry matter amount in the process of feeding the granivores is rather low, the equation constraining the maximum dry matter intake for granivores was still included to assure similarity in approaches to both groups of animals:

$$\sum_{pcon}{}_{(pinput)} \left(pfeed_{cont(pconc,DM)} * PURC_{(pinput)}\right) \leq$$
$$\sum_{gran(head)} \sum_{tech}\left(head\_req_{(head,tech,DM)} * XACT_{(head,tech)}\right) * 1.15,$$

where:

$pconc_{(pinput)}$ - purchased concentrate feed,
$pfeed\_cont_{(pconc,DM)}$ = content of dry matter in purchased concentrate feed [kg/dt],
$PURC_{pinput}$ = amount of purchased inputs [dt] decision variable (see par. 55),
$gran_{(head)}$ = number of ruminant animals decision variable – subset of XACT [LU],
$head\_req_{(head,tech,DM)}$ = feed requirements of dry matter per animal in given technology [kg/LU],
$XACT_{(act,tech)}$ = size of farm activities, [LU] decision variable (see par. 55).

66. **Emissions (constraint)** being the key target for estimations are calculated based on the IPCC methodology (IPCC 2006), with eventual use of their modifications according to the Polish approach implemented by the KOBiZE in preparations of the National Inventory Reports. Key equations regarding the GHG emissions include: 1) enteric fermentation emissions (IPCC equation 10.21, Tier 1+2), 2) manure management direct $CH_4$ (IPCC equation 10.23), 3) manure management direct N emission (IPCC equation 10.25), 4) manure management indirect N losses volatilisation (IPCC equation 10.26), 5) manure management indirect N losses due to leaching (IPCC equation 10.28), 6) emissions from soil - N from crop residuals returned to soils (modified IPCC equation 11.6 [Corrigenda for the 2006 IPCC GLs] ), 7) emissions from urea application (IPCC equation 11.13, Tier 1 method), 8) emissions from agricultural lime application (IPCC equation 11.12, Tier 1 method). The default emission factors for carbon conversion of 0.12 and 0.13 for limestone and dolomite respectively are used according to IPCC (2006).

67. **Enteric fermentation emissions** (Enteric Fermentation IPCC Tier 1+2 Emissions Factors (IPCC equation 10.21) – IPCC 2006b):

$$EF = \left[\frac{GE \times \left(\frac{Y_m}{100}\right) \times 365}{55.65}\right],$$

where:

$EF$ = emission factor (kg $CH_4$ $head^{-1}$ $yr^{-1}$),
$GE$ = gross energy intake (MJ $head^{-1}$ $day^{-1}$),
$Y_m$ = methane conversion factor, per cent of gross energy in feed converted to methane,

The factor 55.65 (MJ/kg $CH_4$) is the energy content of methane.

68. **Manure Management direct $CH_4$** (10.23):

$$EF_{(T)} = (VS_{(T)} \times 365) \times \left[B_{o(T)} \times 0.67\ kg/m^3 \times \sum_{S,k} \frac{MCF_{S,k}}{100} \times MS_{(T,S,k)}\right],$$

where:

$EF_{(T)}$ = annual $CH_4$ emission factor for livestock category T (kg $CH_4$ animal$^{-1}$ yr$^{-1}$),
$VS_{(T)}$ = daily volatile solid excreted for livestock category T (kg dry matter animal$^{-1}$ day$^{-1}$),
365 = basis for calculating annual VS production (days yr$^{-1}$),
$B_{o(T)}$ = maximum methane producing capacity for manure produced by livestock category T ($m^3$ $CH_4$ kg$^{-1}$ of VS excreted),
0.67 = conversion factor of $m^3$ $CH_4$ to kilograms $CH_4$,
$MCF_{(S,k)}$ = methane conversion factors for each manure management system S by climate region k (%),
$MS_{(T,S,k)}$ = fraction of livestock category T's manure handled using manure management system S in climate region k (dimensionless).

69. **Manure management direct N emission** (10.25):

$$N_2O_{D(mm)} = \left[\sum_S \left[\sum_T \left(N_{(T)} \times Nex_{(T)} \times MS_{(T,S)}\right)\right] \times EF_{3(S)}\right] \times \frac{44}{28},$$

where:

$N_2O_{D(mm)}$ = direct $N_2O$ emissions from Manure Management in the country (kg $N_2O$ yr$^{-1}$),
$N_{(T)}$ = number of head of livestock species/category T in the country,
$Nex_{(T)}$ = annual average N excretion per head of species/category T in the country (kg N animal$^{-1}$ yr$^{-1}$),
$MS_{(T,S)}$ = fraction of total annual nitrogen excretion for each livestock species/category T that is managed in manure management system S in the country, dimensionless,
$EF_{3(S)}$ = emission factor for direct $N_2O$ emissions from manure management system S in the country (kg $N_2O$-N/kg N in manure management system S),
S = manure management system,
T = species/category of livestock,
44/28 = conversion of $(N_2O\text{-}N)_{(mm)}$ emissions to $N_2O_{(mm)}$ emissions.

70. **Manure management indirect N losses volatilisation** (IPCC equation 10.26):

$$N_{volatiiizati} = \sum_S \left[\sum_T \left[\left(N_{(T)} \times Nex_{(T)} \times MS_{(T,S)}\right) \times \left(\frac{Frac_{GasMS}}{100}\right)_{(T,S)}\right]\right],$$

where:

$N_{volatilisation\text{-}MMS}$ = amount of manure nitrogen that is lost due to volatilisation of $NH_3$ and $NO_x$ (kg N $yr^{-1}$),

$N_{(T)}$ = number of head of livestock species/category T in the country,

$Nex_{(T)}$ = annual average N excretion per head of species/category T in the country (kg N $animal^{-1}$ $yr^{-1}$),

$MS^{(T,S)}$ = fraction of total annual nitrogen excretion for each livestock species/category T that is managed in manure management system S in the country, dimensionless,

$Frac_{GasMS}$ = percent of managed manure nitrogen for livestock category T that volatilises as $NH_3$ and $NO_x$ in the manure management system S (%).

71. **Manure management indirect N losses due to leaching** (IPCC equation 10.28):

$$N_{leaching-MMS} = \sum_S \left[ \sum_T \left[ \left( N_{(T)} \times Nex_{(T)} \times MS_{(T,S)} \right) \times \left( \frac{Frac_{leachMS}}{100} \right)_{(T,S)} \right] \right],$$

where:

$N_{leaching\text{-}MMS}$ = amount of manure nitrogen that leached from manure management systems (kg N $yr^{-1}$),

$N_{(T)}$ = number of head of livestock species/category T in the country,

$N_{ex(T)}$ = annual average N excretion per head of species/category T in the country (kg N $animal^{-1}$ $yr^{-1}$),

$MS_{(T,S)}$ = fraction of total annual nitrogen excretion for each livestock species/category T that is managed in manure management system S in the country, dimensionless,

$Frac_{leachMS}$ = percent of managed manure nitrogen losses for livestock category T due to runoff and leaching during solid and liquid storage of manure (typical range 1-20%).

72. **Emissions from soil** - N form Crop Residuals returned to soils was generally estimated based on modified IPCC equation 11.6 (Corrigenda for the 2006 IPCC GLs, KOBiZE 2019, p. 177):

$$F_{CR} = \sum_T \{ Crop_{(T)} \times Area_{(T)} \times Frac_{Renew(T)} \times [ R_{AG(T)} \times N_{AG(T)} \times ( 1 - Frac_{Burn(T)} - Frac_{Remove(T)} ) + R_{BG(T)} \times N_{BG(T)} ] \},$$

where:

$F_{CR}$ = annual amount of N in crop residues (above and below ground), including N-fixing crops, and from forage/pasture renewal, returned to soils annually (kg N / yr),

$Crop_{(T)}$ = harvested annual dry matter yield for crop T (kg d.m. / ha),

$Area_{(T)}$ = total annual area harvested of crop T (ha / yr),
$Frac_{Renew(T)}$ = fraction of total area under crop T that is renewed annually,
$RAG_{(T)}$ = ratio of above-ground residues dry matter ($AG_{DM(T)}$) to harvested yield for crop T ($Crop_{(T)}$) (kg d.m. / kg d.m.),
$NAG_{(T)}$ = N content of above-ground residues for crop T (kg N / kg d.m.),
$FracBurn_{(T)}$ = fraction of crop residues burned as indicated in sector 3.F,
$FracRemove_{(T)}$ = fraction of above-ground residues of crop T removed annually for purposes such as feed, bedding and construction (kg N / kg crop-N),
$RBG_{(T)}$ = ratio of below-ground residues to harvested yield for crop T (kg d.m. / kg d.m.),
$NBG_{(T)}$ = N content of below-ground residues for crop (T, kg N / kg d.m.),

T = crop or forage type.

73. **Emissions from urea application** is calculated based on Tier 1 method using equation (IPCC equation 11.13):

$$\mathrm{CO_2\text{-}C}\ Emission = \mathrm{M} \times \mathrm{EF},$$

where:

$CO_2$-C Emission = annual C emissions from urea application (t C/year),
M = annual amount of urea fertilisation [t urea/yr],
EF = emission factor [t C / t urea].

74. **Emissions from agricultural lime application** is calculated according to Tier 1 method using equation 11.12 and the default emission factors for carbon conversion of 0.12 and 0.13 for limestone and dolomite respectively [IPCC 2006]:

$$\mathrm{CO_2\text{-}C}\ Emission = (\mathrm{M}_{limestone} \times \mathrm{EF}_{limestone}) + (M_{dolomite} \times \mathrm{EF}_{dolomite}),$$

where:

$CO_2$-C Emission = annual C emissions from lime application (t C/year),
$M_{limestone}$ = annual amount of calcic limestone ($CaCO_3$) [t/yr],
$M_{dolomite}$ = annual amount of dolomite ($CaMg(CO_3)_2$) [t/yr],
$EF_{limestone}$ = emission factor for limestone – 0.12 [t C / t limestone] [IPCC 2006],
$EF_{dolomite}$ = emission factor for dolomite – 0.13 [t C / t dolomite] [IPCC 2006].

75. Combined GHG emissions from all the aforementioned sources have been used as their total, and therefore served as the basis to constrain emission to an assumed level or to calculate additional taxes or subsidies (depending on applied policy measures) needed to mitigate the emissions.

76. **Maximum size of mitigation measures** are set based on the amount of resources. For area-based GHG mitigation resources it is the area of land available for application of given measures. As afforestation of agricultural land and restoration of the wetlands have long-term effect it is assumed that areas used for these purposes cannot decrease in the following time periods. Therefore:

$$\mathrm{AREAT}_{\mathrm{ghgmit}} \leq \mathrm{AREAT\_MAX}_{\mathrm{ghgmit}}$$

where:

$\mathrm{AREAT\_MAX}_{\mathrm{ghgmit}}$ = maximum size of given GHG mitigation measures, depending on the scenario analysed, and

$$\mathrm{AREAT}_{\mathrm{ghgmit}} \geq \mathrm{AREAT}_{\mathrm{ghgmit}\ \mathrm{t-1}}$$

where:

$\mathrm{AREAT\_MAX}_{\mathrm{ghgmit\ t\text{-}1}}$ = size of mitigation measures applied in the previous period.

77. **Maximum size of agricultural biogas production** is constrained by the amount of manure available at a farm. Thus, the maximal size of XBIO depends on the number and type of animals kept at a farm. It is also assumed that a constructed biogas plant will work at least to the end of modelling period (till 2050), thus the XBIO's capacity should not decrease over time.

$$\mathrm{XBIO} \leq \sum\nolimits_{\mathrm{head}_{(\mathrm{act})}} \sum\nolimits_{\mathrm{tech}} (\mathrm{XBIO}_{(\mathrm{act,tech})} * \mathrm{XACT}_{(\mathrm{act,tech})})$$

where:
$\mathrm{XBIO}_{(\mathrm{act,tech})}$ = maximum capacity for a Livestock Unit of farm animals in a given technology.

$$\mathrm{XBIO} \geq \mathrm{XBIO}_{\ \mathrm{t-1}}$$

where:
$\mathrm{XBIO}_{\ \mathrm{t\text{-}1}}$ = capacity of biogas generation in the previous period.

## 2.6. Calibration

78. A typical linear programming model suffers from significant inaccuracy in reproduction of the baseline year values. The theoretical optimum determined on the basis of the baseline year differs from the reality observed (Heckelei and Britz 2005). Thus linear models require to be calibrated by adding various types of restrictions. Most often these are so-called crop rotation restrictions, specifying the maximum or minimum share of individual crops. In addition to the often weak theoretical or empirical justification for such restrictions, in case of model construction for farm aggregates (e.g., for the FADN farm type), they excessively limit the scope of acceptable solutions for simulated scenarios (Ciaian et al. 2013). In order to eliminate this phenomenon, the EPICA model

uses Positive Mathematical Programming (PMP). For the first time, the PMP approach was formalised and described in the work by Howitt (1995). However, in earlier studies in regard to substantiation of expert opinions supporting policy-making similar techniques were successfully used (Howitt and Gardner 1986, Kasnakoglu and Bauer 1988, Schmitz 1994). Based on these findings a new technique was added to existing linear models as a substitute for numerous calibration restrictions.

79. The idea of the PMP method is generally based on the use of dual prices binding calibration constraints, which ensure mapping of the observed reality in the linear model (stage 1). Obtained dual prices are used to determine the parameters of the non-linear objective function in a way that reflects the observed reality, and real levels of decision variables without introduction of additional restrictions (stage 2). Stage 1 of the above procedure can be formally described as follows:

$$\max Z = p'x - c'x$$

$$Ax \leq b \; [\lambda]$$

$$x \leq (x^0 + e) \; [\rho]$$

$$x \geq 0,$$

where:
$p$ = (N×1) vector of product prices,
$x$ = (N×1) vector of production activity levels,
$c$ = (N×1) vector of accounting cost per unit of activity,
$A$ = (M×N) matrix of coefficients in resource constraints,
$b$ = (M×1) vector of available resource quantities,
$x^o$ = (N×1) vector of observed activity levels,
$\varepsilon$ = (N×1) vector of small positive numbers,
$\rho$ = dual prices for calibration constraints,
$\lambda$ = (M×1) vector of dual variables associated with the resource constraints.

80. The addition of a calibration constraint vector will provide an accurate representation of the level of observed values $\mathbf{x}^o$ by the linear model presented above. Assuming, of course, that the limitations and resource balances contained in the model allow such a solution (Hazell and Norton 1986). Accurate reproduction of the observed levels of values in this case means compliance within a range of not more than $\mathbf{x}^o$ to $\mathbf{x}^o+\varepsilon$.. The use of the sum of $\mathbf{x}^o+\varepsilon$ allows the binding nature of all restrictions arising from the amount of resources available and thus prevents the loss of validity of their dual prices.

81. The vector of variables $\mathbf{x}$ can be divided into two parts: ((NM)×1) vector of “preferred” activities, $\mathbf{x}^p$ being limited by calibration constraints, and vector (M×1) of “marginal”

activities, $\mathbf{x}^m$ being limited by available resources. To simplify, let's assume that all elements of the $\mathbf{x}^o$ vector are nonzero and all resource constraints are binding. Then, based on the Kuhn-Tucker theorem, we assume that:

$$\rho^p = p^p - c^p - A^{p\prime}\lambda$$

$$\rho^m = [0]$$

$$\lambda = (A^{m\prime})^{-1} (p^m - c^m).$$

82. Indices **p** and ***m*** denote subsets of vectors and matrices belonging to the preferred and marginal variables, respectively. Dual prices resulting from calibration restrictions for marginal variables are 0 and for preferred variables they are equal to the difference in price and marginal cost determined as the sum of unit costs and marginal costs of resources used ($\mathbf{A}^{p\prime}\lambda$). It should be noted here that in such a model dual prices resulting from resource constraints are conditioned by the profitability of marginal activities.

83. In the second stage of building the model using the PMP, the dual prices of calibration constraints for the preferred variables $\rho^p$ are used to specify a non-linear objective function such that the marginal costs of preferred activities are as much as their prices at the observed level of activity in the baseline year $\mathbf{x}^o$. Assuming the appropriate shape of the non-linear function used (convex in the range of the level of activity) the solution to the problem created will be "the limit point, which is a combination of binding restrictions and first order derivative conditions" (Howitt 1995).

84. In principle, any non-linear function meeting the above condition can be used in the 2nd stage of model calibration. Due to the simplicity of calculation and the lack of strong arguments against it, in most cases the square function of costs is used. The general form of this function is shown below:

$$C^v = d'x + 1/2x'Qx,$$

where:

d = (N×1) a parameter vector related to the linear element of the cost function,

Q = (N×N) a positively symmetrical matrix of parameters related to the square element of the cost function.

85. The parameters **d** and **Q** are then set so that the marginal cost (MCV) meets the following conditions:

$$MC^v = \frac{\partial C^v(x^0)}{\partial x} = d + Qx^0 = c + \rho.$$

86. It should be noted here that the derivative of the variable costs function does not include the opportunity costs ($\mathbf{A}^{p\prime}\lambda$), which were captured in the primary (linear) model in the form of prices of dual resource constraints. Assuming that the parameters **d** and **Q** meeting the above conditions have been estimated, a non-linear optimisation problem

can be formulated, and the solution without additional restrictions will be equal to the observed levels of activity:

$$maxZ = p'x - d'x - 1/2x'Qx$$

$$Ax \leq b\ [\lambda]$$

$$x \geq 0.$$

87. It should be emphasised that at this stage the dual prices resulting from resource constraints in model at point $\mathbf{x}^o$ do not differ from those set in model. They are still determined by the marginal profitability of marginal operations at point $\mathbf{x}^{om}$ and can be set at $(\mathbf{A}^{m}')^{-1}\ [\mathbf{p}^m - (\mathbf{d}^m + \mathbf{x}^{om}\mathbf{q}^m)]$, which equals $(\mathbf{A}^{m}')^{-1}[\mathbf{p}^m\text{-}\mathbf{c}^m]$ at the stage of specifying d and Q parameters. Therefore, the value of equation $\lambda$ **=** $\mathbf{(A^m\ ')^{-1}\ (p^m\ -\ c^m)}$ remains unchanged.

## 2.7. Limitations

88. While the supply part of the model was prepared for a wide range of farm types, yet due to lack of available data it considers only two production technologies: intensive and extensive. Varying breakdown of these two technology shares within each of the considered farm activities allows to differentiate to some extent the technologies used at different farm types. However, there are other factors, as the used production technologies might also depend on the scale of production, and this is not fully reflected in the applied dataset.

89. The PMP technique allows perfect calibration of the model regarding the baseline year conditions. However, the Howitt PMP method might lead to reduction of model response to the assumed shocks in case of those activities which have a marginal share in the baseline year. Further development of the calibration techniques applied in the model is required to achieve both exact calibration and more realistic responses of marginal farm activities in analysed scenarios.

# 3. Market module

## 3.1. Introductory remarks

90. The goal of the market module is to predict how changes in the supply of agricultural products induced by climate policy affect the equilibrium (or market) prices of these products. The module, therefore, is thought to work in tandem with the farm module: the latter predicts changes in supply given the prices, while the former predicts changes in prices given change in supply.

91. The market module is a partial equilibrium model: the set of prices of agricultural products is derived from equilibrium conditions that equate demand and supply for every such product. In practice, the model starts by setting the relation between supply and prices (the supply curve) and the relation between demand and prices (the demand curve). Changes of supply predicted by the farm module are used to shift the supply curve. This shift leads to a new equilibrium with new set of prices. This information is then re-entered in the farm module which again predicts changes in supply. The iteration between the two modules continues until price convergence is obtained.

92. The demand curves are consistent with the micro-founded demand system (i.e., derived from the optimisation problem of representative consumer in national economy). As such, it is aligned with the principles of microeconomic theory. Therefore, the model is robust to pitfalls of some numerical models that derive their predictions from the economic patterns of the past not taking into account that those patterns evolve over time together with the changes in the environment of economic actors (the Lucas critique). The derivations are based on the assumption of rational behaviour and rational expectations of economic agents.

93. The full implementation of the micro-founded economic model would however require putting additional computational burden when solving the numerical model, which could result in significant increase in the computation time. To avoid this problem, we derive the mathematical predictions of the model and introduce the resulting relations (so called, reduced form model) as equations in the code. The mathematically rigorous discussion and derivation of these equations together with its calibration strategy is outlined in the technical note that supplements this document.

94. The derivation of the demand system follows closely the approach in the CAPRI model. The most significant change with respect to CAPRI model is that we allow the choice of total expenditure on agricultural products to be a part of consumer optimisation problem. This implies that total expenditure on agricultural products is endogenously determined by the model. It also implies that spending on agriculture commodities does not need to add up to a constant, which implies that there is no need to impose any restrictions on elasticity of demand for each commodity and we can use its estimate directly from empirical literature.

95. The demand system is derived in two steps: first, the domestic consumer decides on the demand for each agricultural good and the demand for external (non-agricultural) good. Next, the domestic consumer decides on the composition of this good: what share will be purchased from domestic and foreign producers. The final demand for domestic producers is composed of the demand from domestic consumer and the demand of foreign consumer (export). We assume that the latter demand is a simple log-linear function of domestic prices.

96. The module does not require any input information on the prices of products. This feature is obtained by setting appropriate normalisation of prices: the unit of the value of each product in the model is the value of that product in the baseline scenario. The output of the model is the percentage change of the price with respect to the baseline scenario.

97. The module requires setting two types of parameters: share parameters and elasticities. Share parameters are computed using the shares of each product in total value of production, values of import and export. The share of production statistics were taken from the farm module baseline scenario. The import and export statistics were taken from Statistics Poland (GUS 2019). The elasticity statistics were taken from the own and cross-price elasticities tables in the CAPRI model. The detailed strategy for calibration of the elasticities is outlined in the technical note that supplement this document.

98. Due to availability of data we limited the number of products in the market module to 10 commodities: i) sugar beet and potatoes, ii) fruits and vegetables, iii) cereals, fodder crops and other crops, iv) oils, v) cattle for meat, vi) dairy cattle, vii) pigs, viii) poultry for eggs, ix) poultry for meat, and x) other animals. There is an additional linking module that aggregates the results from the farm module before they enter the market module and disaggregates the results of the market module before they enter the farm module.

99. A separate set of commodities included in the market module are those that are used (or purchased) in another sector. In the current version of the model this set contain only one element: biomass, which can be used in the energy sector. For this set of commodities, the demand curve is not derived from representative consumer utility function – instead it is assumed that the demand curve takes a linear form and that its parameters are calibrated using the output of other sectoral model. In the case of biomass, the parameters of the demand curve are calibrated in the iterative process using the simulations of the MEESA model.

## 3.2. Deriving the demand system

### 3.2.1. Choice between commodities

100. We define *n* to be the number of agriculture commodities in the market module and *N*=*n*+1 to be the total number of commodities in the module. The additional good is an external good.

101. To predict the equilibrium changes in prices and quantities the model must specify the set of demand functions: $\boldsymbol{x}(\boldsymbol{p}, y)$, where $\boldsymbol{x}$ is a vector $(x_1, \dots, x_n, x_e)$, $x_i$ is the demand for commodity $i$, $x_e$ is the demand for the external (non-agriculture good), $\boldsymbol{p}$ is the vector of commodity and external good prices and y is the total expenditure of the consumer.

Since the market module takes into account changes in the partial equilibrium, we assume that y is fixed and does not depend on the policies considered.

102. We assume that the vector of demands $\boldsymbol{x}(\boldsymbol{p}, y)$ is generated by a rational representative agent who maximises utility. We also assume that utility function is continuous and the consumer's preferences are not satiated. By assuming the existence of representative consumer we abstract from the problems of aggregation (see e.g., the literature on almost ideal demand systems, Deaton and Muellbauer (1980)).

103. One possibility to derive such demands is to first assume some functional form for direct utility function and then derive the demand conditional on prices and income from the first order conditions to consumer maximisation problem. This approach, however, raises several difficulties. The most commonly used direct utility function: Constant Elasticity of Substitution have too few parameters to appropriately simulate the cross-price effects. For instance, in the case of CES functions, there is only one parameter that describes elasticity of substitution between goods. Therefore it does not allow to take into account that some pairs of goods are characterised with higher elasticity of substitution than others. For instance, after an increase in price of beef, consumers might be more willing to substitute beef with pork then beef with wheat.

104. Therefore, following the literature on demand systems (e.g., Deaton and Muellbauer (1980), Ryan and Wales (1996)), we take different approach: deriving the consumer demands from assumed expenditure function. By the theory of duality, for every consumer maximisation problem there is an analogous minimisation problem whose solution contains all information about the consumer preferences. The solution of minimisation problems consists of two elements: the vector of Hicksian demand (i.e., demand as a function of vector of prices and assumed utility level u): $\boldsymbol{h}(\boldsymbol{p}, u)$ and expenditure function, $\boldsymbol{y}(\boldsymbol{p}, u)$ which describes minimum expenditure required to achieve a given level of utility. By duality, for every expenditure function which is (i) non-decreasing in $p_i' s$ and $u$, (ii) homogenous of degree one in prices, (iii) concave in $\boldsymbol{p}$ and (iv) continuous in $\boldsymbol{p}$, there must be some minimisation problem and associated utility maximisation problem that would generate it. Moreover, the information on $\boldsymbol{y}(\boldsymbol{p}, u)$ is sufficient to recover the demand system generated by that maximisation problem. In fact, by Hotelling lemma, Hicksian demand for good *i* could be derived by differentiating the expenditure function with respect to price of that good.

105. In the literature, the most common demand systems derived from expenditure functions are translog, Almost Ideal Demand System and Generalised Leontief. All three systems allow to specify a separate cross-price elasticity for each pair of goods. Although AIDS seems to be the most consistent framework (since it ensures the demand could be derived from aggregating individual demand), we decided to opt for Generalised Leontief to preserve the compatibility of our model with the CAPRI agricultural model.

However, contrary to the system in CAPRI model we assume the presence of external good - we discuss the reasons and implications of this change in assumptions later on.

106. The classical demand system based on Generalised Leontief is derived by assuming that the expenditure function takes the form

$$y(\boldsymbol{p}, u) = \frac{-\sum_k \sum_i b_{ki} p_k^{0.5} p_i^{0.5}}{u}.$$

This could be immediately transformed into an indirect utility function.

$$u(\boldsymbol{p}, y) = \frac{-\sum_k \sum_i b_{ki} p_k^{0.5} p_i^{0.5}}{y}.$$

Following Hotelling lemma, the Hicksian demand, could be derived by taking the derivative of expenditure function with respect to price.

$$h_i(\boldsymbol{p}, u) = \frac{-\sum_k b_{ki} p_k^{0.5} p_i^{-0.5}}{u}.$$

Substituting the indirect utility function in the place of $u$ (following the theory of duality), we can derive the Marshalian demand:

$$x_i\big(\boldsymbol{p}, u(\boldsymbol{p}, y)\big) = \frac{-\sum_k b_{ki} p_k^{0.5} p_i^{-0.5}}{\sum_k \sum_i b_{ki} p_k^{0.5} p_i^{0.5}} y.$$

107. This simple system is characterised by $N$ parameters, which describe both, shares of each commodities and own- and cross-price elasticity of substitution. Consequently, in this simplest Generalised Leontief system, if one matches the data on elasticities there is not enough degrees of freedom to match the data on shares. For this reason, we follow Ryan and Wales (1996; the same approach is followed in CAPRI model) to assume a slightly modified expenditure function

$$y(\boldsymbol{p}, b) = \sum_i d_i p_i - \frac{\sum_k \sum_i b_{ki} p_k^{0.5} p_i^{0.5}}{u}. \quad (1)$$

Following the same derivations as before, we can derive Marshalian demand as:

$$x_i(\boldsymbol{p}, y) = d_i + \frac{\sum_k b_{ki} p_k^{0.5} p_i^{-0.5}}{\sum_k \sum_i b_{ki} p_k^{0.5} p_i^{0.5}} \left(y - \sum_i d_i p_i\right).$$

Parameter $d_i$ is a location parameter for the demand curve. It could be interpreted as part of consumption of commodity $i$ which is independent of changes in prices and changes in income.

This system is characterised by $N * N + N$ parameters and thus allows to match, both shares and elasticities data during the calibration phase.

108. In addition to the commodities purchased by a representative consumer, the model includes commodities purchased and used by other sectors: energy and transport. All variables and parameters associated with this set are labelled using a $bio$ index. For these commodities the demand curve takes a linear form:

$$x_{bio}(\boldsymbol{p}) = a_{bio}^{demand} - b_{bio}^{demand} p_{bio}.$$

In the current version of the model only one such commodity is included: biomass that is consumed by the energy sector.

109. Parameters of the demand curve are set separately for each year for each simulations by exchanging information with the MEESA model. The exchange of information takes an iterative form: in the first step, the MEESA model simulation generates information on the volume of demand for a given price. In the second step, the EPICA utilises this information to recalibrate $a_{bio}^{demand}$ parameter and adjust the position of the demand curve. In the third step, the EPICA simulation generates information about the new price. In the fourth step this information is fed back to the MEESA model, closing the loop. The iteration continues until convergence is achieved.

### 3.2.2. Trade effects

110. In the previous subsection, we derived the demand for commodity $i$ from the optimal choices of consumer. We argue that our approach is equivalent to representing he preferences of consumers between commodities using a utility function $u(\boldsymbol{x})$ and then finding the vector of $x_i'$ that maximise that utility. We interpret $x_i$ as a consumption of commodity $i$. However, as it will be described later, this consumption is not in physical units.

111. Consumption of commodity $i$ can be satisfied by two types of goods: those that are produced domestically ($q_{di}$) and those that are imported ($q_{mi}$). If we assumed that the consumption $x_i$ is measured in physical units, we would need to assume that $x_i = q_{di} + q_{mi}$, which would impose that domestic and imported products are perfect substitutes. Instead, following Armington assumption, we describe the preferences between these two types of products with the CES aggregation:

$$x_i = \left( (\gamma q_{di})^{\frac{\xi-1}{\xi}} + ((1-\gamma) q_{mi})^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}}, \tag{2}$$

where $\xi$ is the elasticity of substitution and $\gamma$ is the share parameter.

112. Note also that while $x_i$ does not have an empirical counterpart, $p_i x_i$ does have. By the definition of expenditure function used in the previous subsection, $p_i x_i$ stand for the total spending the representative consumer devotes for good $i$. Note also that

$$p_i x_i = p_{di} q_{di} + p_{mi} q_{mi}. \tag{3}$$

113. Under these assumptions we can now state the complete optimisation problem of a consumer. The consumer must choose quantities $q_{mi} q_{di}$ for each commodity $i$ to maximise utility for a given set of prices, $p_{di}$ and $p_{mi}$, and given income, $y$ subject to $u = u(\boldsymbol{x})$ and (2).

114. The optimisation problem could be solved in two stages. First, for every possible vector $x$, we find the optimal choice of quantities $q_{mi}, q_{di}$ for a given vector of prices and income. By the theory of duality, this optimal choices could be found from the expenditure minimisation problem. The solution of the problem defines the optimal relation between expenditure for domestic and imported goods as well as the minimum expenditure necessary to attain $x_i$. That expenditure function defines the price for acquisition of $x_i$. Second, given the vector of $p_i$ and total expenditure, $y$ the consumer chooses optimal choices.

115. Under our assumptions described above, the solution to the first problem takes the form:

$$\frac{p_{di}q_{di}}{p_{mi}x_{mi}} = \left(\frac{p_{di}/\theta}{p_{mi}/(1-\theta)}\right)^{1-\sigma}, \quad (4)$$

and

$$p_i = \frac{p_{di}q_{di}+p_{mi}x_{mi}}{x_i} = \left((p_{di}/\theta)^{1-\sigma} + \left(p_{mi}/(1-\theta)\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \quad (5)$$

$$x_i(\boldsymbol{p}, y) = d_i + \frac{\sum_k b_{ki} p_k^{0.5} p_i^{-0.5}}{\sum_k \sum_i b_{ki} p_k^{0.5} p_i^{0.5}} (y - \sum_i d_i p_i). \quad (6)$$

116. These three equations together with (3) determine the optimal demand for $q_{di}$ as a function of $p_{di}$ and $p_{mi}$. To understand the logic of the model mechanism, consider a change in the price of domestically produced commodity $i$, $p_{di}$, due to changes on supply side. Equation 5 translates this change into a change in price index for $x_i$. Then equation (6) allows to project change in $x_i$. The resulting change in expenditure for commodity $i$, $p_i x_i$ is split into changes of expenditure on domestic and imported commodities using (3) and (4). Information on the change in expenditure on domestic goods $i$ allows to recover the change in $q_{di}$.

117. To close the model, we need to specify the demand for exported good. We assume that foreign consumers (indexed with $f$) perform similar optimisation choosing between goods that are supplied by producers in their countries ($q_{fi}$) and those that were exported to their countries ($q_{xi}$). The solution to their problem implies

$$\frac{p_{fi}q_{fi}}{p_{xi}q_{xi}} = \left(\frac{p_{fi}/\gamma}{p_{xi}/(1-\gamma)}\right)^{1-\xi}. \quad (7)$$

118. We assume that foreign consumers' choices of $q_{fi}$ are exogenous, i.e. does not depend on the prices of domestic goods. In addition, we assume that the price of good exported is proportional to the price of domestic goods: $q_{di} \propto q_{di}$. This assumption allow us to fully characterise deviations of $q_{xi}$ as a function of deviations in $p_{di}$. The total change in demand for domestically produced goods $i$ is the sum of a change in $q_{di}$ and $q_{xi}$.

## 3.3. Set-up of the demand system

### 3.3.1. Normalisations

119. Both, the input and the output of the model are expressed as a deviation from the baseline, i.e. a scenario in which the intervention does not take place. Appropriate normalisations implies that the model does not require any information on the physical quantities or monetary values. The only information required for the calibration of the model are shares in spending and elasticities of demand (both unitless). We discuss those normalisations below.

120. First we normalise total expenditure of the representative consumer to unity:

$$y = \sum_i x_i p_i = 1.$$

Thus all possible expenditures that appear in the mode: $x_i p_i$, $p_{di} q_{di}$, $p_{mi} q_{mi}$, $p_{fi} q_{fi}$ and $p_{xi} q_{xi}$ are expressed in terms of units of total expenditure. This normalisation is permitted since, as in every partial equilibrium model, number of equations is larger than the number of variables. In our case we have 5xN +1 equations (3)-(7) and $y = \sum_i x_i p_i$) and 5xN variables ($q_{di}'s$, $q_{mi}'s$, $q_{xi}'s$, $x_i's$ and $p_i's$).

121. Second, we normalise the price of good $x_i$ in the baseline to unity, $p_i^{baseline} = 1$ for every $i$. Thus an equilibrium price, $p_i$ shall be interpreted as 1 + percentage deviation from the state when the change in intervention does not take place.

122. Third, note from equations (3)-(7) that we actually do not require information on $\theta$, since the percentage deviation of $p_{di}/\theta$ is exactly the same as the percentage deviation in $p_{di}$. For formal clarity we define several new variables and substitute them in the place of prices and quantities of domestic and imported goods. Table 1 describes this substitution.

**Table 1. Substitution**

| New variable | Definition |
|---|---|
| $\tilde{\boldsymbol{p}}_{di}$ | $p_{di}/\theta$ |
| $\tilde{\boldsymbol{p}}_{mi}$ | $p_{mi}/(1-\theta)$ |
| $\tilde{\boldsymbol{q}}_{di}$ | $q_{di}\theta$ |
| $\tilde{\boldsymbol{q}}_{mi}$ | $q_{mi}(1-\theta)$ |
| $\tilde{\boldsymbol{q}}_{xi}$ | $q_{xi}\theta$ |

Source: CAKE/KOBiZE own study

### 3.3.2. Model equations

123. Using these normalisations we can restate the set-up of the model on the demand side. The final set-up consists of five equations described briefly below.

124. Equation

$$p_i^{1-\sigma} = \tilde{p}_{di}^{1-\sigma} + \tilde{p}_{mi}^{1-\sigma} \tag{8}$$

is a counterpart of equation (5). It determines price of good *i* for consumers in the domestic market.

125. Equation

$$x_i = d_i + \frac{\sum_k b_{ki} p_k^{0.5} p_i^{-0.5}}{\sum_k \sum_i b_{ki} p_k^{0.5} p_i^{0.5}} (y - \sum_i d_i p_i) \tag{9}$$

is a counterpart of equation (6) determines domestic demand of good *i* as a function of prices. Equation (9) together with equation (8) determines total domestic expenditure on good *i*.

126. Equation

$$\frac{\tilde{p}_{di}\tilde{q}_{di}}{\tilde{p}_{mi}\tilde{q}_{mi}} = \left(\frac{\tilde{p}_{di}}{\tilde{p}_{mi}}\right)^{1-\sigma} \tag{10}$$

is a counterpart of equation (4). It determines the ratio of expenditures on domestically produced and imported goods as a function of prices.

127. Equation

$$p_i x_i = \tilde{p}_{di}\tilde{q}_{di} + \tilde{p}_{mi}\tilde{q}_{mi} \tag{11}$$

is a counterpart of equation (3). It determines the relation between total expenditure, expenditure on domestically produced goods and value of import. Equation (8) together with equation (9) allow to decompose changes in total expenditure on good I to changes in expenditure on domestically produced goods and changes in value of import.

128. Equation

$$\tilde{p}_{di}\tilde{q}_{xi} = \tilde{p}_{di}^{1-\xi}\Omega_i \tag{12}$$

is a counterpart of equation (7). It determines value of export as a function of domestically produced goods. $\Omega_i = \left(\frac{1-\theta}{p_{fi}(1-\gamma)/\gamma}\right)^{1-\xi} p_{fi} q_{fi}$ is an exogenous constant[8], which can be interpreted as value of export when price of domestically produced goods is equal to unity.

129. The final equation that close the model is equilibrium clearing, i.e. an equation that equate domestic supply with total demand for domestically produced products:

[8] For numerical reasons, in the code of the model the equation 10 has been rearranged to $(\tilde{p}_{di}\tilde{p}_{mi})^{1-\sigma} = p_i^{\sigma-1}(\tilde{p}_{di}^{1-\sigma} + \tilde{p}_{mi}^{1-\sigma})$ and equation 9 to $\tilde{q}_{di}\tilde{p}_{di}^{\sigma} = \tilde{q}_{mi}\tilde{p}_{mi}^{\sigma}$.

$$\tilde{q}_{si} = \tilde{q}_{di} + \tilde{q}_{xi}\,, \qquad (13)$$

where $\tilde{q}_{si} = q_{si}\theta$ and $q_s$ is the total supply of commodity $i$ by domestic producers.

Due to restrictions on the parameter values, we also need to normalise some of the parameters $d_i$ and $b_{ik}$, however we leave this until the section on calibration.

## 3.4. Supply side

130. The set of supply curves derived from the profit maximisation under the assumption of quadratic cost curve:

$$\pi = p_{di}q_{si} - a_s q_{si} - \frac{b}{2}q_{si}^2,$$

where $p_{di}$ is the price of commodity $i$ by domestic suppliers, $q_{si}$ is the quantity supplied and $a_s$ and $b_s$ are parameters. The solution gives

$$\tilde{p}_{di} = \tilde{a}_s + b_s\tilde{q}_{si}.$$

## 3.5. Calibration

131. In this section we discuss which values should be assigned to the parameters of the model. We discuss separately the calibration of the share parameters and elasticities.

### 3.5.1. Share parameters

132. Due to the normalisations in section 3.3 there are only three share parameters for each commodity $i$: $d_i$ (location parameter for the demand curve; see equation 1), $\Omega$ (value of export when price of domestic good is equal to unity) and $p_{mi}$ (price of import). In order to calibrate $d_i$, we first have to normalise the level of indirect utility in the baseline. We assume that $u^{baseline} = -N$, where $N$ is a number of commodities. Since we normalised $y = 1$ and $p_i = 1$ for every $i$ and since we can (as we discuss in section on calibration of elasticity parameters) normalise $\sum_i b_{ki} = 1$ for every $k$, equation 1 implies that $\sum_i d_i = 0$. Thus, using 11, we find that in the baseline

$$SPEND_i = x_i^{baseline} = d_i + \frac{1}{N},$$

where $SPEND_i$ denotes the share of total expenditure devoted to spending for commodity $i$. This allows to calibrate $d_i$ for every commodity.

133. Since we assume that $p_{mi}$ is constant, $p_{mi} = p_{mi}^{baseline}$ for every simulation. $p_{mi}^{baseline}$ can be calibrated using equations (9) and (10), ratio of value of domestic to imported

consumption of commodity $i$ obtained in the data and the normalisation $p_i^{baseline} = 0$. Using these relations we can establish

$$p_{mi}^{baseline} = (1 + DOMIMP_i)^{\frac{1}{\sigma-1}},$$

where $DOMIMP$ is the ratio of domestic to imported value of consumption in the baseline which is recovered from the data.

134. Using the (10), we can also calculate the level of $\tilde{p}_{di}$ in the baseline, which will help us in computing $\Omega$:

$$\tilde{p}_{di}^{baseline} = DOMIMP_i^{\frac{1}{1-\sigma}} \tilde{p}_{mi}^{baseline}.$$

Then we can recover $\Omega$ from equation (12):

$$\Omega_i = EXP_i \left(\tilde{p}_{di}^{baseline}\right)^{\xi-1},$$

where $EXP_i$ is the ratio of export to total disposable income of the consumers (recalling that that income is a numeraire).

### 3.5.2. Supply parameters

135. In the market module there are only two supply side parameters to be estimated: $\tilde{a}_s^{baseline}$ and $b_s$.

136. Parameter $\tilde{a}_s^{baseline}$ could be estimated as follows: at the baseline $p_i^{baseline} = 1$, so from equation 8 we can recover $\tilde{q}_{di}^{baseline}$:

$$\tilde{q}_{di}^{baseline} = \frac{1}{\tilde{p}_{di}^{baseline}} \left[\frac{SPEND_i}{1+(DOMIMP_i)^{-1}}\right].$$

Next, we can recover the baseline supply of each commodity:

$$\tilde{q}_{si}^{baseline} = \tilde{q}_{di}^{baseline} + \frac{EXPORT}{\tilde{p}_{di}^{baseline}}.$$

Given the assumed for of supply curve, $\tilde{a}_s^{baseline} + b_s \tilde{q}_{si}^{baseline} = \tilde{p}_{di}$, finding the value of $\tilde{q}_{si}^{baseline}$ allows to calibrate $\tilde{a}_s^{baseline}$:

$$\tilde{a}_s^{baselin} = \tilde{p}_{di} - b_s \tilde{q}_{si}^{baseline}.$$

137. Parameter $b_s$ could be estimated from the elasticity of supply predicted by the supply module (SM). Specifically, since

$$\frac{dq_s}{dp_d}\frac{p_d}{q_s} = \frac{1}{b_s}\frac{p_d}{q_s}.$$

It must be that

$$b_s = \frac{1}{\epsilon_i^{supply}} \frac{p_d^{baseline}}{q_s^{baseline}}.$$

However, in this version of the model we follow a simpler approach and assume $b_s = 1$. Note that choice of $b_s$ affects the speed of convergence in the iteration between farm and market modules, however it does not affect final results.

In this version of the model we assume $b_s = 1$. Note that choice of $b_s$ affects the speed of convergence in the iteration between farm and market modules, however it does not affect final results.

138. The supply curve is recalibrated for every iteration with the supply module. The change in the $a_s$ could be computed as follows: totally differentiating the log of supply curve relation in the neighbourhood of the baseline:

$$d \ln \tilde{a}_s = \frac{-b_s}{\tilde{p}_{di}^{baseline} - b_s \tilde{q}_{di}^{baseline}} d \ln q_{si}.$$

Thus,

$$a_s = a_s^{baseline} - b_s \left( \frac{q_{si}}{q_{si}^{baseline}} - 1 \right),$$

where $DEVSUP_i = \frac{q_{si}}{q_{si}^{baseline}} - 1$ is the percentage change in supply of commodity $i$ due to intervention predicted by the supply module. $b_s$ could be computed from projections of the supply module.

139. Exactly the same procedure is applied for the commodities purchased by a representative consumer and for the commodities purchased in other sector (set $bio$).

### 3.5.3. Trade elasticities

140. We use the values of elasticities for the trade effects that were suggested in CAPRI model documentation (Britz and Witzke 2014, see the table on page 166). To our knowledge there are no estimates of trade elasticities specific to Polish economy available in the literature.

### 3.5.4. Price elasticities

141. Parameters $b_{ik}$'s determine own and cross price elasticities in the model and thus should be calibrated in order reflect the elasticities estimated from the data. First note, that according to the microeconomic theory (by the properties of the Hessian matrix), the cross-price elasticities must be symmetric. Thus if we define $\epsilon_j^k = \frac{dq_k}{dp_j} \frac{p_j}{q_k}$, then $\epsilon_j^k = \epsilon_k^j$. Also, due to law of demand, own price elasticity must be negative, $\epsilon_j^j$. We correct the estimated elasticities manually to satisfy this criteria.

142. Under symmetry the number of those parameters is given by $\frac{n(n-1)}{2}$. The number of own and cross- price elasticities for N goods is also given by $\frac{n(n-1)}{2}$. However, exact

identification of parameters $b_{ik}$ with elasticities estimated from the data is not possible. In the following subsection we explain why the system with $\frac{n(n-1)}{2}$ parameters and $\frac{n(n-1)}{2}$ conditions matching model and data cannot be solved.

#### 3.5.4.1. The problem of mapping elasticities and the parameters

143. In this subsection we provide a mathematical argument explaining why estimates of elasticities cannot be matched one-to-one with the slope parameters of demand curves in the model ($b_{ik}$).

144. We start the derivations by simplifying the equations describing the demand curve. To this end we define $F \stackrel{\text{def}}{=} \sum_i d_i p_i$, $G \stackrel{\text{def}}{=} \sum_i \sum_j b_{ij}\sqrt{p_i p_j}$, $G_j \stackrel{\text{def}}{=} \frac{dG}{dp_j} = \sum_i b_{ij}\sqrt{\frac{p_i}{p_j}}$, $G_{jj} \stackrel{\text{def}}{=} \frac{d^2G}{dp_j^2} = \frac{-1}{2}\sum_{i\neq j} b_{ij} p_j^{-1.5} p_i^{0.5}$ and $G_{ij} \stackrel{\text{def}}{=} \frac{d^2G}{dp_i dp_j} = \frac{1}{2} b_{ij} p_j^{-0.5} p_i^{-0.5}$. Note that we introduce these definitions to ease the exposition of our mathematical derivations. The economic interpretation of these expressions is not relevant at this point.

The definitions above allow us to describe the demand as:

$$x_i = d_i + \frac{G_i}{G}(y - F).$$

145. Next, we evaluate own and cross-price elasticities of demand using the definitions above. The derivative of quantity demanded with respect to own price is given by

$$\frac{dq_j}{dp_j} = \frac{G_{jj}}{G}(y - F) - \frac{G_j^2}{G^2}(y - F) - \frac{G_j}{G} d_j,$$

if we evaluate it at the baseline (recalling that $p_i^{baseline} = 0$, which also implies that $F^{baseline} = 0$ and multiplying by $\frac{p_j}{q_j}$, we can express own elasticity of demand as

$$\epsilon_j^j = \frac{\frac{G_{jj}}{G} - \frac{G_j^2}{G^2} - \frac{G_j}{G} d_j}{d_j + \frac{G_j}{G}}.$$

The derivative of quantity demanded with respect to cross price is given by

$$\frac{dq_k}{dp_j} = \frac{G_{jk}}{G}(y - F) - \frac{G_j G_k}{G^2}(y - F) - \frac{G_k}{G} d_j.$$

Again, if we evaluate it at the baseline and multiply with $\frac{p_j}{q_k}$ we can express cross elasticity of demand as:

$$\epsilon_j^k = \frac{\frac{G_{jk}}{G} - \frac{G_j G_k}{G^2} - \frac{G_k}{G} d_j}{d_k + \frac{G_k}{G}}.$$

146. Now, consider summing $\epsilon_k^j \frac{q_k}{q_j}$ across $k$'s:

$$\sum_k \left[\epsilon_k^j \frac{q_k}{q_j}\right] = \frac{1}{q_j} \sum_k \left[\frac{G_{jk}}{G} - \frac{G_j G_k}{G^2} - \frac{G_k}{G} d_j\right].$$

When we evaluate this at the baseline the expression is equal to unity. Since at the baseline, $q_j$ and $q_k$ are fixed (must exactly match the data), this implies a restriction on every row in the nxn matrix of elasticities.

147. The intuition behind this restriction is straightforward: since consumer must spend all its income on $n$ goods and that income is fixed, the changes in expenditures for the goods induced by a change in price of good $j$ must sum up to zero.

As a result the exact matching of $n$ empirically estimated elasticities in each row of the elasticities matrix with $n$ corresponding elasticities predicted by the model is not possible.

#### 3.5.4.2. Solution to the matching problem in EPICA

148. In EPICA we adopt the following solution to this this problem: in each row of the matrix we match exactly $n$ empirically estimated elasticities for agricultural goods with $n$ elasticities predicted by the model and let one elasticity (the one corresponding to the change in external good) to adjust to satisfy the restriction.

This leaves us with the system of $\frac{N(N-1)}{2}$ parameters $b_{ik}$ and $\frac{N(N-1)}{2} - N$ conditions for matching them with the data. To identify the parameters we therefore need to assume a normalisation for parameters in each row of the $b_{ik}$'s matrix. Specifically, we normalise $\sum_i b_{ij} = 1$ . This also implies that at the baseline, and $G = n$ for every $j$.

### 3.5.5. Aggregating elasticities

149. We recover the empirical estimates of elasticities from the CAPRI model. However, those elasticities are defined for a very narrowly defined commodities. The commodities in the market module in our model is broader and aggregate several commodities in the CAPRI (2014). In this section we discuss how we compute own and cross elasticities for our commodities from the estimates provided by the CAPRI. We do it using an example:

Suppose that in CAPRI model there are two types of grains: wheat $(N)$ and rye $(R)$. Suppose that in our model we have only one commodity of grain $(G)$ that aggregates the two commodities in the CAPRI model. Note that the expenditures in commodities in CAPRI must sum up to expenditure for grains in our model: $p_G G = p_W W + p_R R$. Own-price elasticity for grains ($G$) is given by:

$$\epsilon_{p_G}^G = \frac{\partial \ln p_G G}{\partial \ln p_G} = \frac{p_W W}{p_G G}\frac{\partial \ln p_W W}{\partial \ln p_G} + \frac{p_R R}{p_G G}\frac{\partial \ln p_R R}{\partial \ln p_G},$$

which simplifies to:

$$\epsilon_{p_G}^G = s_W \frac{\partial \ln p_W W}{\partial \ln p_G} + s_R \frac{\partial \ln p_R R}{\partial \ln p_G},$$

where $s_W = \frac{p_W W}{p_G G}$ and $s_R = \frac{p_R R}{p_G G}$ are the share of the two commodities in the aggregated commodity.

150. Note that the effect, $\frac{\partial \ln W}{\partial \ln p_G}$ could be decomposed into two effects: one driven by a change in price of wheat and another one driven by a change in price of rye. That is, $\frac{\partial \ln W}{\partial \ln p_G} = \frac{\partial \ln W}{\partial \ln p_W} + \frac{\partial \ln W}{\partial \ln p_B}$. We assume that the change of all prices within the group is the same, that is $\partial \ln p_G = \partial \ln p_W = \partial \ln p_R$. Consequently

$$\epsilon_{p_G}^G = s_W\left(\epsilon_{p_W}^W + \epsilon_{p_R}^W\right) + s_R\left(\epsilon_{p_W}^R + \epsilon_{p_R}^R\right).$$

151. Regarding cross price elasticities, suppose that we are interested in the effect of price change in grains on meat ($M$, composed of beef $B$ and pork $P$). This can be now stated as

$$\epsilon_{p_M}^G = \frac{\partial \ln G}{\partial \ln p_M} = \frac{\partial \ln G}{\partial \ln W}\frac{\partial \ln W}{\partial \ln p_M} + \frac{\partial \ln G}{\partial \ln R}\frac{\partial \ln R}{\partial \ln p_M}.$$

152. The quantity, $G$ could be computed as $G = \frac{p_W W + p_R R}{p_G}$. By duality $p_G$ is a function of $p_W$ and $p_R$ and parameters of utility function, thus it is independent of changes in $p_M$. Then

$$\frac{\partial \ln G}{\partial \ln W} = \frac{p_W W}{p_W W + p_R R} = s_W.$$

Hence we arrive to

$$\epsilon_{p_M}^G = s_W\left(\epsilon_{p_B}^W + \epsilon_{p_P}^W\right) + s_R\left(\epsilon_{p_B}^R + \epsilon_{p_P}^R\right).$$

## 3.6. Limitations

153. The modelling of trade effects in EPICA is limited. The model considers only two regions: Poland and the rest of the world (ROW). It does take into account that the ROW demand for agriculture commodities produced in Poland (i.e. export from Poland) as well as ROW supply (i.e. import to Poland) depends on changes in prices and costs of production of these commodities in Poland. However, the model is unable to predict which countries specifically are involved in this trade. Moreover, it does not take into account that climate policy could affect costs of production and commodity prices in the ROW. If these costs increase, the prices for consumers in Poland will increase more than what is suggested by the model currently. In addition, this would lead to smaller drop in production in Poland and, likely, smaller reduction in GHG emissions.

# 4. Reporting of results

154. The EPICA model consists of two modules: farm and market. Therefore the modules are solved iteratively until results given by each model converge.

155. The main result of the farm module is an optimal structure of the production activities, which provides the highest farm income in conditions described in the scenario for each modelled farm type. It means that the most basic solution of the model includes:

a) number of hectares of crop activities for each of the crops;

b) number of livestock units for each considered animal activity, both distinguished according to applied technologies;

c) farm income achieved at optimal structure of farm activities.

Apart from the optimisation model results a number of indicators are also being calculated. In so-called "post modelling phase", which includes calculations after solver's completion, a number of indicators characterising organisation and economic performance of farm are calculated, based on the optimised production structure. The most important indicators provided by the model are:

- farm income per hectare of arable land, which allows the comparison of farm economic performance between the farm types;
- production of commodities in physical units produced in each of considered scenarios by each of the farm types;
- value of produced commodities for each of the farm types calculated at the assumed or optimal (depending whether the market model is used) prices;
- amount of purchased inputs, as the mineral fertilisers or animal feedstock by category of input and farm type, both in physical and value terms;

- full balances of crop nutrients and animal feed purchased from the market in optimal scenario;
- detailed data on GHG emissions from agricultural sector with division according to emission sources and emitted gases for each of the farm types (e.g., enteric fermentation - $CH_4$, manure management - $CH_4$ and $N_2O$, soil management $N_2O$, liming and urea application – $CO_2$). The model also provides results for avoided emissions due to the introduced LULUCF activities. In line with the IPCC regulations those emissions are reported as LULUCF.

156. However to achieve results of the farm level module it is needed to provide expected price changes under each of assumed scenarios. The market model provides potential market reaction. Thus the main results of the market module are:

- relative changes of prices due to changes of supply resulting from farm activities;
- changes in demand of agricultural commodities under considered scenarios;
- changes of international trade of agricultural commodities.

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# Annex 1: Clasification of farms types & acitivities

### Table 2. Classification of farm types

| Farm types by specialisation | Farm size classes | Farm type codes |
|---|---|---|
| Cereals | Small, medium, large | Cer_S, Cer_M, Cer_L |
| Crops | Small, medium, large | Cro_S, Cro_M, Cro_L |
| Cattle | Small, medium, large | Cat_S, Cat_M, Cat_L |
| Pigs | Small, medium, large | Pig_S, Pig_M, Pig_L |
| Mixed | Small, medium, large | Mix_S, Mix_M, Mix_L |
| Other | Small, medium, large | Oth_S, Oth_M, Oth_L |
| Semi-subsistence | N/A | Ssub |

Source: CAKE/KOBiZE own study

### Table 3. Classification of farm activities

| Farm activities | Farm activity code |
|---|---|
| Wheat | WHEA |
| Other cereals | OCER |
| Oilseeds | OILS |
| Sugar beets | SUGB |
| Potatoes | POTA |
| Proteins (grain) | PROG |
| Proteins (fodder) | PROF |
| Maize (grain) | MAIG |
| Maize (silage) | MAIF |
| Fruits (short term <5 years) | FRUS |
| Vegetables (short term <5 years) | VEGE |
| Fruits & Vegetables (>5 years) | FRUL |
| Permanent grassland | PGRA |
| Grass on arable land and other fodder crops | GROA |
| Fallow Land - Ecological Focus Area | FALL |
| Other crops | OCRO |
| Energy crops | ECRO |
| Cattle for beef | CATT |
| Dairy cattle | DAIR |
| Pigs for meat | PIGS |
| Poultry for meat | POLM |
| Poultry for eggs | POLE |
| Other animals | OANI |

Source: CAKE/KOBiZE own study

# Annex 2: Main changes in the EPICA model between versions

Main changes in the EPICA model between current 2.0 version and previously published 1.0 version:

- Distinguishing between commodities purchased by representative consumers and commodities purchased by other sectors.
- Description of demand for commodities purchased by other sectors.
- Description of iterative strategy to link supply and demand for commodities purchased by other sectors.
- Description of applied GHG mitigation measures in the agricultural sector (e.g., agri-biogas production).
- Supplementing the model with LULUCF-based GHG mitigation measures (e.g., restoration of wetlands and afforestation of agricultural land).
- Providing additional activities and outputs regarding the amount of biomass for energy purposes produced by farming sector.